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A math research center may be set up in a corner of a room, preferably with a chalk board. Children are able to create many of the games, charts and stories to be used in such a center. The center may be supplemented with books, games, blocks, poppit beads, counters, checkers, and number line charts.

Attention is given to a wide variety of methods, materials and ideas intended to make mathematics more meaningful to the student. Different types of number lines may be used in all grades. In one example children are given tags numbering from 0 to 10. Five are asked to step forward. The children are asked how many more are needed to make six (one steps forward) and so forth.

Poppit jewelry which can make many different combinations of colors may also be used. Playing checkers or the use of checkers and a board may serve the same purpose.

"Numo" and "Numa" are small animated characters (like Jiminy Cricket) invented by a first grade teacher to help the children learn numbers. They are placed on a number line interspersed with letters that spelled "NUMO". The children are asked how many jumps are needed to make the sequence spell his name. Another example of techniques presented is the use of hand puppets to ask children number questions.

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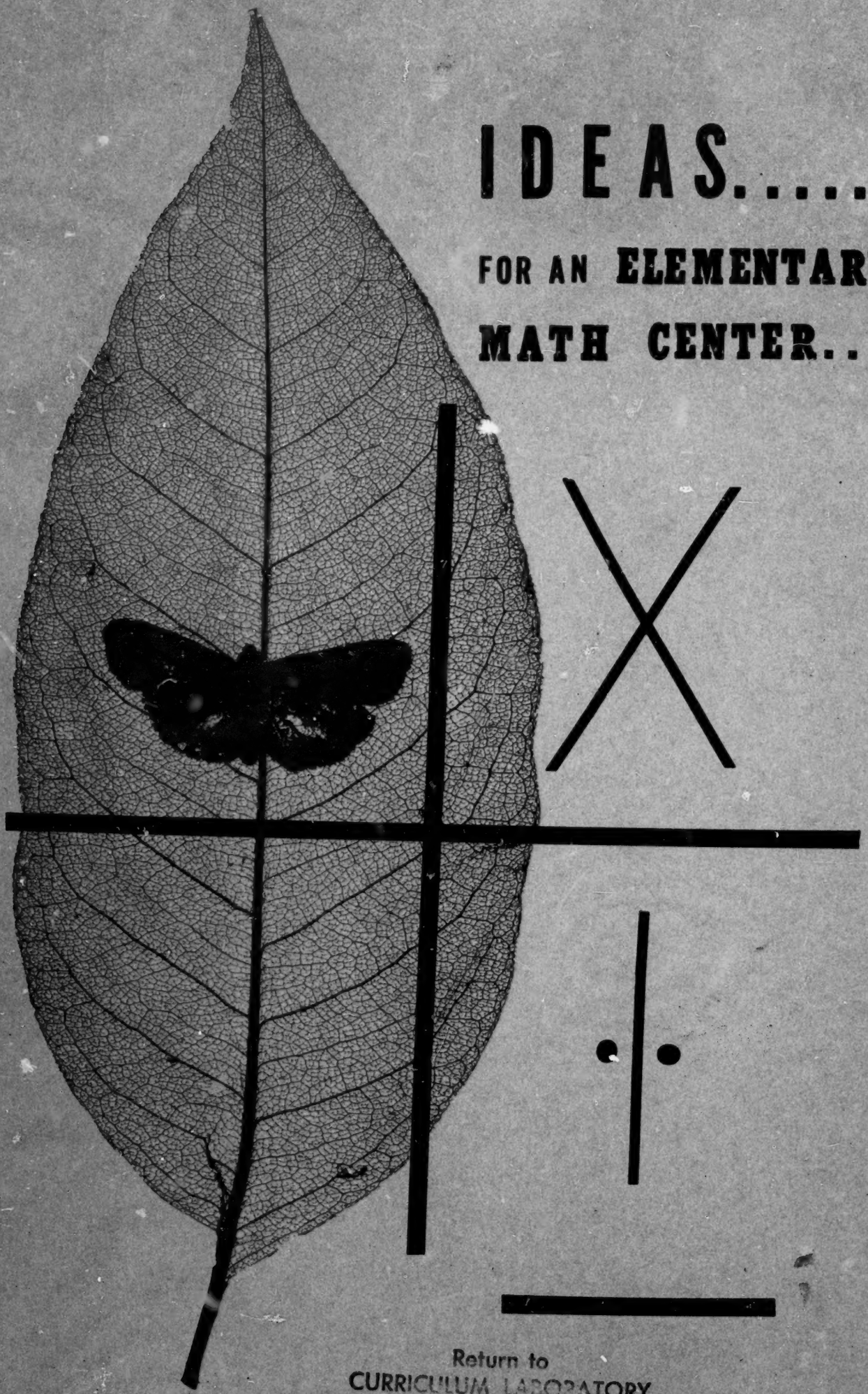
IDEAS.....

FOR AN ELEMENTARY MATH CENTER....



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CAN YOU MAKE A PICTURE TO SUGGEST....?

A Manual of Ideas & Materials
for a Contemporary Classroom
MATH RESEARCH CENTER

Prepared by a Workshop of the
Bucks County School Study Council
Bucks County, Pennsylvania
Summer, 1961

Cover by:
Henry Ray

Asst. Superintendent
Bucks County Schools

Published by
BUCKS COUNTY SCHOOL STUDY COUNCIL
50 North Main Street
Doylestown, Pennsylvania

FOREWORD

I extend my congratulations to the members of the Bucks County staff and to Bucks County supervisors and teachers who developed the materials and ideas in this manual during the Elementary Mathematics Workshop held in the summer of 1961.

A great amount of educational experimentation concerns school organizational patterns. The new MATH WORKSHOP FOR CHILDREN is exemplary of the changes which must take place in the curriculum itself. Curriculum changes only as content changes, and as teachers are able to change and adapt their methods. This summer's workshop represents the genuine efforts of a professional group to acquaint themselves with new curriculum content in arithmetic, and with techniques for bringing it into the educational diet of children.

Your efforts and energies in helping to keep the Bucks County Schools in the forefront in the field of curriculum development are genuinely appreciated and greatly valued.

George E. Raab
Superintendent
Bucks County Schools

October, 1961
Bucks County, Pa.

INTRODUCTION

There's magic in mathematics! But many of us never believed this until the summer of 1961, when we attended the Elementary Mathematics Workshop sponsored by the Bucks County School Study Council at Council Rock High School in Newtown, Pennsylvania. Thanks to Bob Wirtz and Mort Botel, co-authors of the MATH WORKSHOP FOR CHILDREN, many of us became pretty excited about numbers.

The ideas and materials described here are the result of that stimulation, and were developed by primary teachers who saw the possibility of improving early arithmetic teaching through the use of manipulative materials. Although many of these ideas are basically for use with young children, there are valuable suggestions for every elementary school classroom.

An area designated as a MATH RESEARCH CENTER will create interest, enthusiasm, laboratory facilities, and incentive for your students. You can provide rich and varied experience with numbers and mathematical patterns as you investigate questions that "go somewhere," such as: How many names has a number? ... Can we keep a record of what's happening? ... Who has another idea? ... Can you make a picture to suggest.....?

Since young children lack the lingual and reading ability to comprehend or reproduce complicated mathematics, these ideas can be embedded in gamelike experiences. "Big ideas" do not have to wait for competence in reading and speaking. Activities in which children discover the "rules of the game" and can "change the rules of the game" lay the ground work for real understanding of numbers, improvement of traditional attitudes about arithmetic, and better transfer of learning.

Traditionally, a fact has been the answer to a number problem; children were taught facts as though they were the end product, rather than a means to more discovery. In the early years, children learn about distance and relationships by first feeling and then seeing. As all of the senses are more fully developed, they move from the concrete to the abstract. Puzzles, score keeping, manipulative media, pictures, charts, and interesting materials serve as early means of communication to help children feel the movement, never-ending patterns, fun and challenge of mathematics. But the most important catalyst is a creative teacher whose own attitude reflects her enthusiasm for academic pursuits.

As you try some of these ideas and materials with your students, share the ones that work best with your students with your colleagues. Above all, let ideas come - for they always bring others with them.

We can raise children's creative and thinking levels as we inspire a growing edge of knowledge. Those of us fortunate enough to be using the Wirtz, Botel MATH WORKSHOP FOR CHILDREN, and who have had the inspiration of working with the co-authors right here in Bucks County, will be in the vanguard of the "new" mathematics for contemporary living, teaching, and learning.

Elizabeth S. Wright
Curriculum Specialist
Bucks County Schools

October, 1961

Participants in the Elementary Mathematics Workshop and contributors to this manual:

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Murray Cohen	-Bristol Twp.
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Alta Leary	-Centennial
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Eleanor Zemel	-Centennial
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Organizing a .....

..... MATH RESEARCH CENTER

Your MATH RESEARCH CENTER could be a couple of tables and a book case for storing all kinds of mathematics laboratory equipment, games, and books with number themes.

If possible, locate these collections near either a chalk board or a bulletin board area. Often children will make charts, original games, or mathematical Headline Stories which can be displayed in the center.

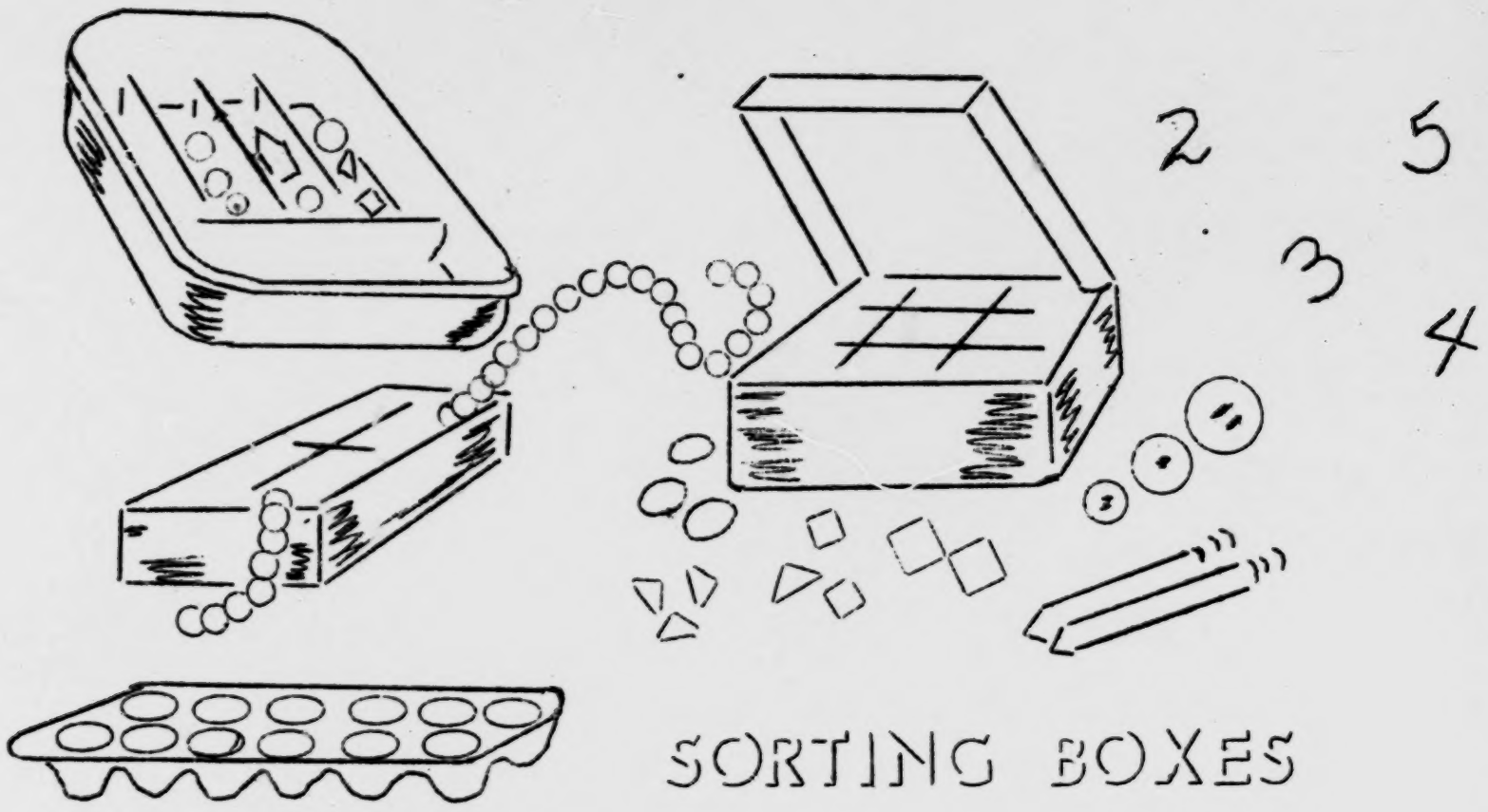
A round, octagonal or rectangular library table and several chairs enable groups of five or six children to work together or on individual "research."

The book case could house interesting stories about numbers, such as those published by the Golden Book Company; or How Big is Big?; or All Kinds of Time; or the Numo Books which will be published this year by Encyclopedia Brittanica Films, Inc; or The Story of Time and Measurement; or Space in Your Future. Have children bring counting books and number stories from home. Let them try writing some original stories to add to the collection. Games which come in boxes can also be stored here.

In addition to the standard equipment we usually think of such as rulers (the reversible ones with inches on one side and quarter and half inches on the two edges of the opposite side are excellent for young children), clock faces with both Roman and Arabic numerals, gallon, quart, pint, and half-pint containers, a yard stick, Cuisinierre Rods or Stern arithmetic materials, blocks or cubes, and play money - consider the following:

... Parquetry Design Blocks (squares, triangles, diamonds, etc.)
... Plastic Numbers and Relations Kits
... Thermometers - Speedometers - Pedometers - Altimeters
... Number Boards and Strips
... Number Line Charts
... Poppit Beads and Standard Bead Frames
... An Abacus
... Balancing Scales
... Sorting Boxes, containing many different numbers and kinds of manipulative objects
... Discs, counters, colored tooth picks, pick-up sticks
... Checkerboards and checkers. Blank paper discs of same size as checkers
... Extra Basic Work Forms (Wirtz, Botel; Encyclopedia Brittanica Films, Inc.)

You'll be amazed how the collection will grow as children make unusual contributions such as an old gas or water meter!



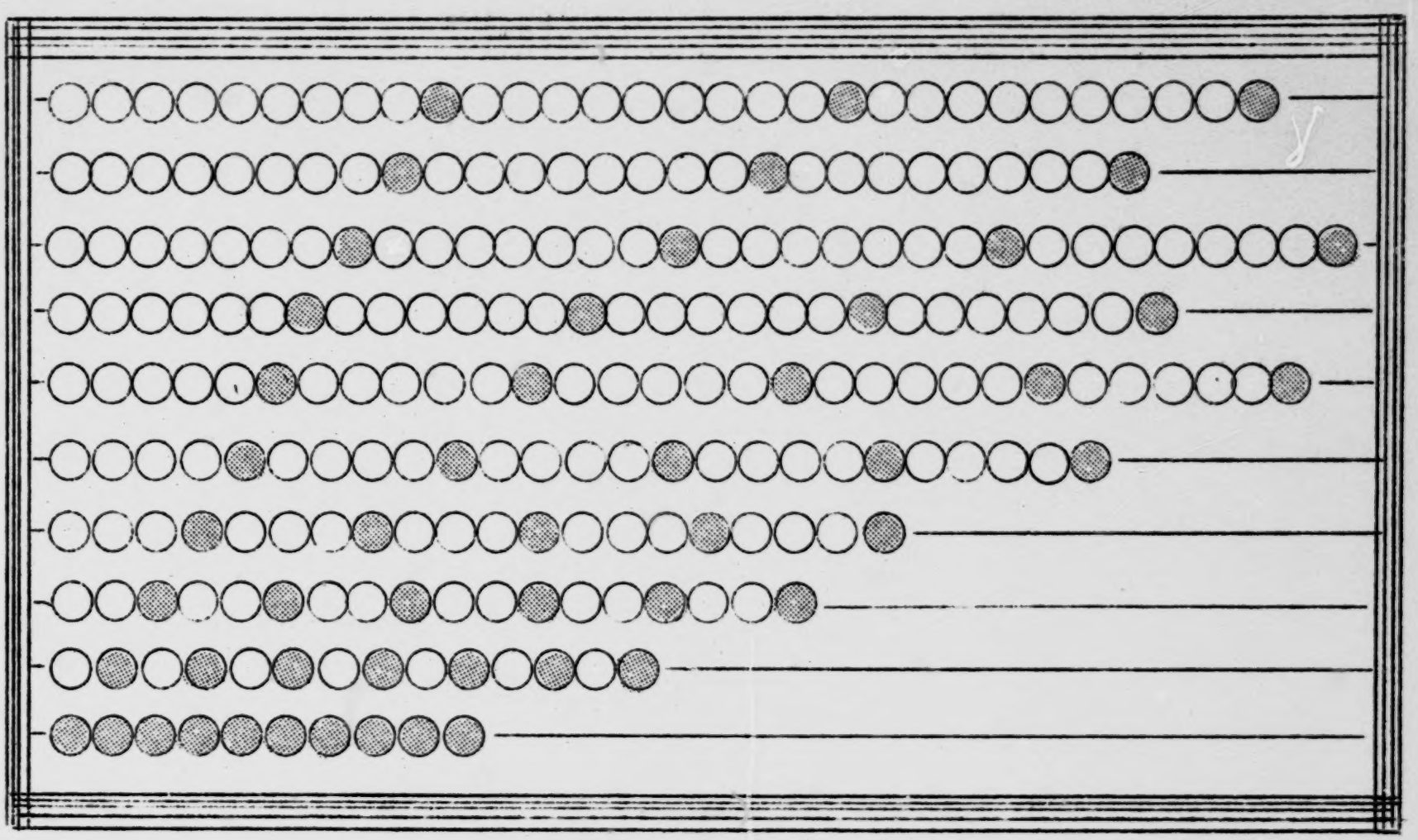
SORTING BOXES

Manipulative materials help our students discover, probe, think, question, and search for patterns and structure. Ask your students to collect all sorts of small objects for both Math Research Center and individual Sorting Boxes. Sorting boxes may be egg cartons, muffin tins, silver trays, jewelry boxes, cigar boxes, etc.

Collect objects differing in shape, size, color, and number. These may be discs, buttons, colored toothpicks, toy planes, cars, or boats in matching pairs, beads, checkers, beans, cardboard strips, crayons, pencils, erasers, and geometric shapes.

There are many possible ways to group objects, not only by increasing number, but also by differences and similarity in shape, size, and color. Children not involved in a classroom activity at the moment may keep records of "What's going on" on their desk tops or in their own sorting boxes. Objects can be moved from compartment to compartment, regrouped or arranged according to the "rules of the game." Big ideas can be expressed without complicated language.

Right from the beginning, children can express thinking which is very basic to all subject-matter areas and to the very essence of pure mathematics. Clear, graphic forms help answer such questions as: How many names has a number? How can we show? Who has an idea? Can we make a picture to suggest.....?



AN UNUSUAL COUNTING FRAME

If you have a tens frame on hand, you can use it for many rich experiences such as those suggested in Poppit Arithmetic and "How many names for a number?". Just see the patterns pop out when you put beads in multiples of 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10, on a different kind of counting frame.

Parents could help their children make one of these and the whole family would have fun with this kind of homework. Those old beads you've been saving work very well; string them on wire and anchor them on each side with carpet tacks. An old picture frame serves very nicely, or a frame might be a good shop project. Row one needs 10 beads of the same color. The other rows need two colors, one for the basic number and a different color for the multiple.

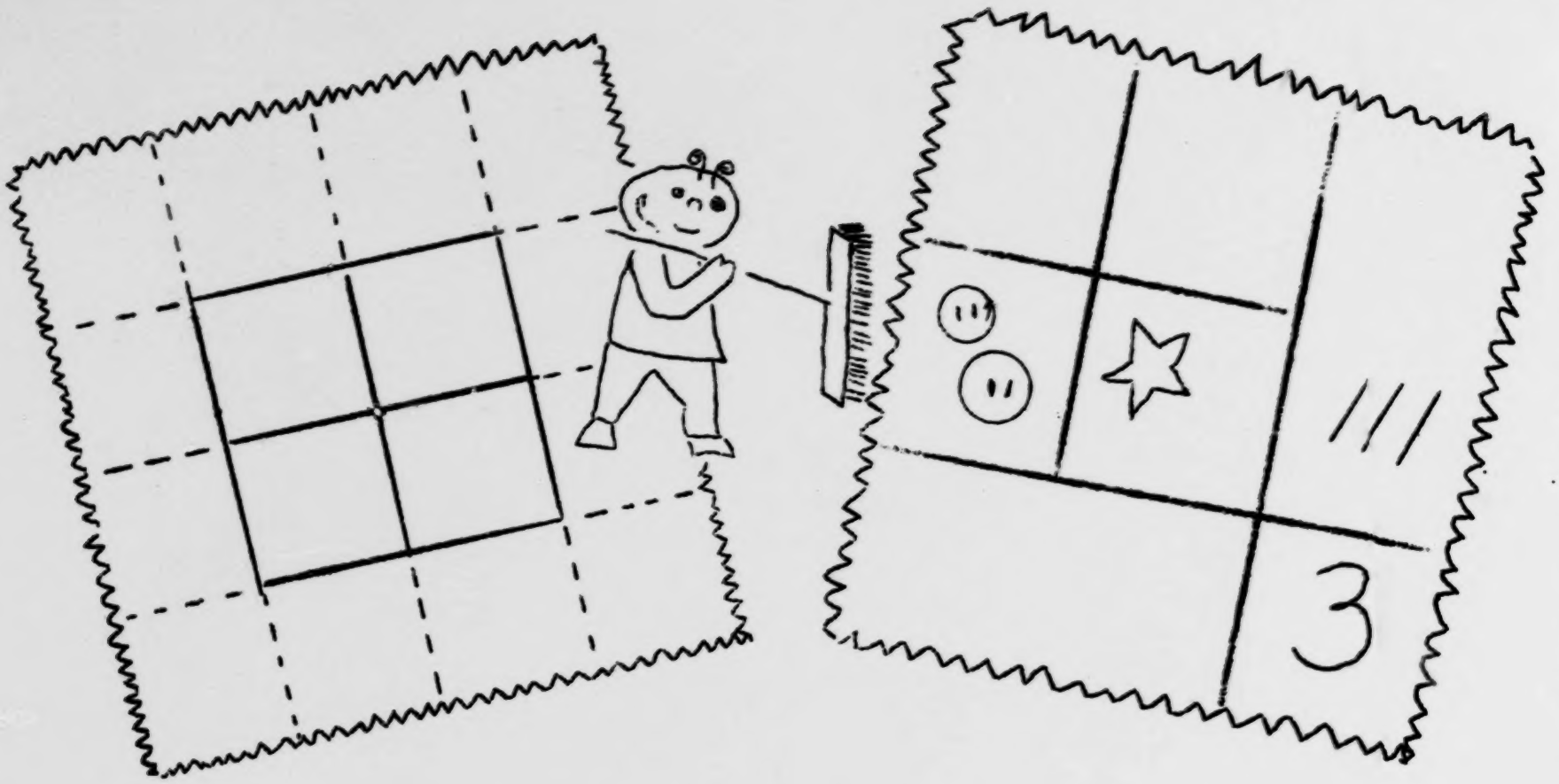
With a counting frame like this, students can make a pictorial record of number combinations and discoveries, ranging from the simple to the complex. Addition, subtraction, multiplication, and division are right at our fingertips as easily as the beads are moved about on the wire. Children see and feel the infinite possibilities for arranging numbers.

+	1	2	3	4
1	2			
2	3	4		
3	4	5	6	
4	5	6	7	8

−	4	4	4	4
9	5	5		
8	4	4	4	
7	3	3	3	3
6	2	2	2	2

X	4	4	4	4
1	4			
2	8	8		
3	12	12	12	
4	16	16	16	16

÷	1	2	4	8
4	4	2	1	½
8	8	4	2	1
12	12	6	3	1½
16	16	8	4	2



OILCLOTH BASIC WORK FORMS

These may be used in many ways. Children can place objects on them, "sweep" them over and down, and count them, tally them with marks, and eventually use numbers. Numbers of "things" feel and look better on a shiny, colorful surface.

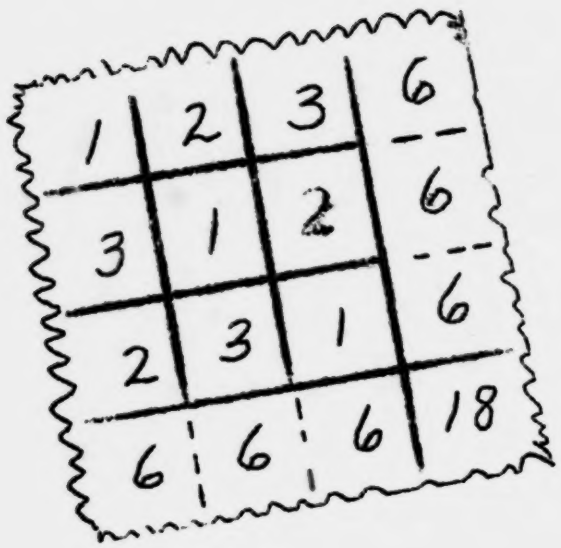
Two forms are shown here, but the possibilities are as endless as the many workforms and their infinite patterns and uses. MAY BE USED AT ALL LEVELS.

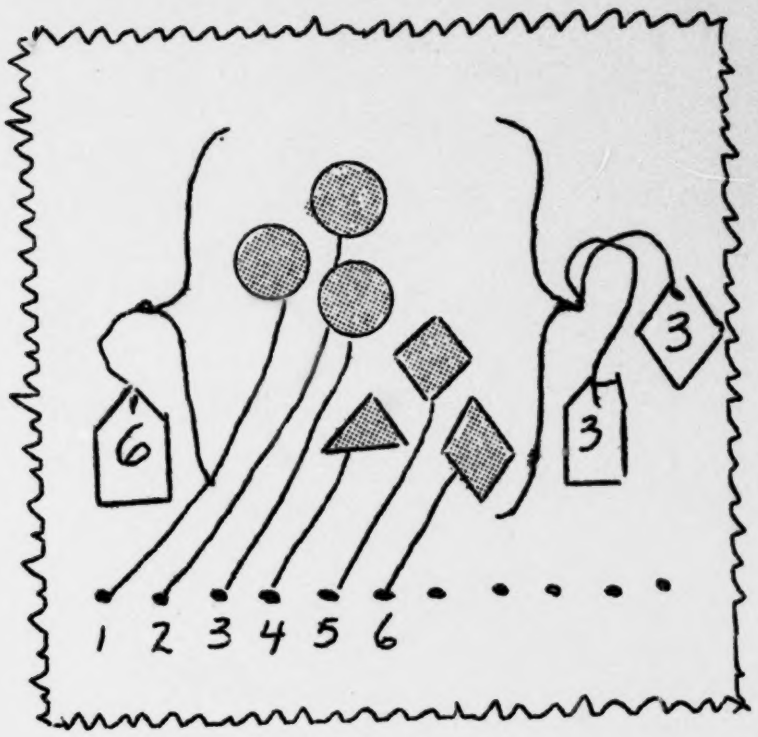
<u>To make these, you need</u>:

1. Yellow oilcloth - plastic or regular will do.
2. Black magic marker or India ink.

<u>Directions for making</u>:

For each square you wish to make, you will need a piece of material, 12" x 12". Cut outside edges with pinking shears; this keeps them from tattering and helps them lay flat. Measure and pencil in Basic Work Form you decide to use. With a magic marker, draw heavy lines with broad side and light lines with pointed end. The grease pencil is for working on the form. The form is permanent, but grease pencil marks can be wiped off. When not in use, roll them over a paper towel roll, and secure with rubber band.





Basic Work Forms are

Adaptable to MANY

Levels of Difficulty . . . . .

Start out with one or two oilcloth basic work forms; as you and the children work with them and become more familiar with their use, add others. The two basic forms above also present rich possibilities for manipulating numbers.

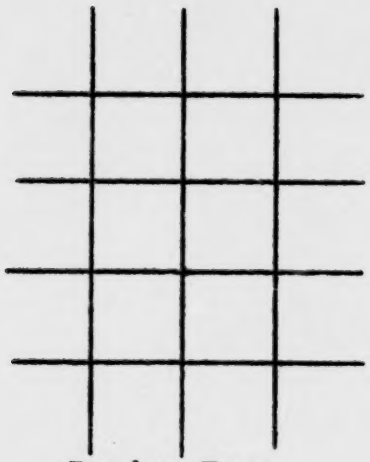
These forms of "Cross Number Puzzles" are limited first to addition and subtraction, later can be used for multiplication and division. They serve to point out the fundamental laws of arithmetic, while providing rich kinaesthetic experience with "What's happening here?".

Children from first grade on are fascinated with them. They like to see if they can make one that "won't work" as they discover the fundamental structures of mathematical ideas and manipulations long before they know the names for them.

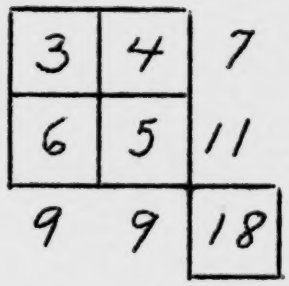
Study the Basic Work Forms to see how they might lend themselves to "Oilcloth Arithmetic." Add to your oilcloth squares collection as another medium of versatility in mathematics research, for these mathematically rich forms help pupils explore ideas in greater depth.

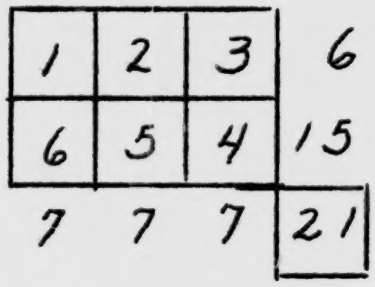
MORE BASIC WORK FORM IDEAS

Reusable forms may be made from a large piece of oak tag, covered with a sheet of clear plastic and thumbtacked to a bulletin board. The plastic can be labeled with marking crayon or grease pencil and erased with cleansing tissue. This technique can be used for many kinds of charts and graphs, and also makes it possible to use one large chart marked into squares for many kinds of teaching-learning situations and record keeping.



Basic Form



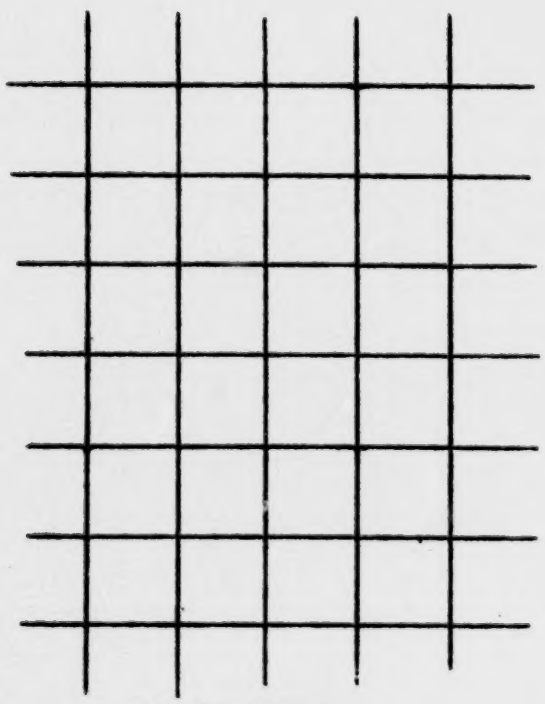


etc.

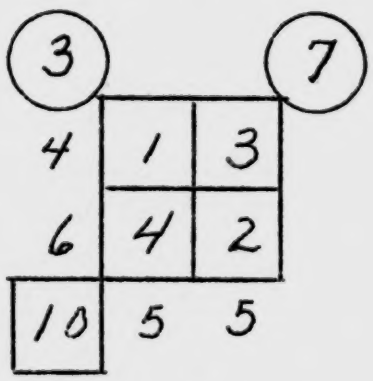
Adaptations

These illustrations show one squared basic form which, covered with plastic, can be adapted to many uses such as: the addition and subtraction ladder, the adding machine, the multiplying machine, cross number puzzles for adding, subtracting, multiplying or dividing, and record keeping of all kinds.

-- Marguerite Glenn Warwick



Basic Form

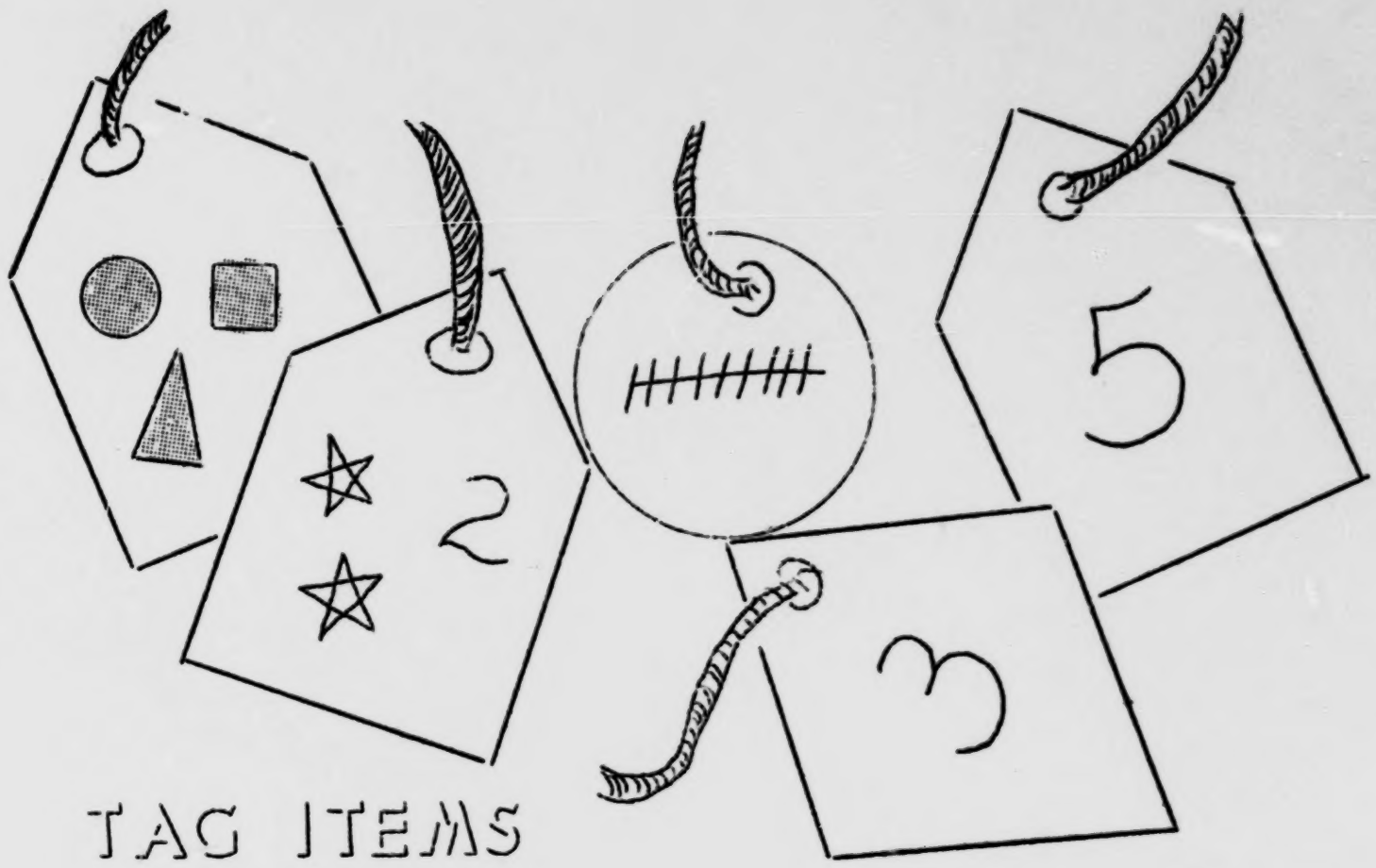


7 + 3 = 10
5 + 5 = 10
4 + 6 = 10
3 + 7 = 10, etc.

X	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

2 × 1 = 2
2 × 2 = 4
2 × 3 = 6
2 × 4 = 8
3 × 1 = 3, etc.

Some Adaptations



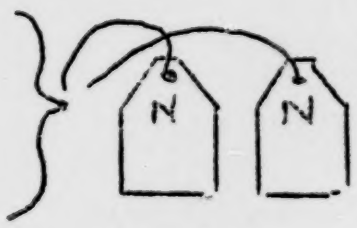
TAG ITEMS RIGHT FROM THE START

Construct tags of cardboard or oak tag. A notebook reinforcement will serve nicely to circle the hole through which the string is tied. Sometimes you will want to use hooks, instead of string, for easy removal.

For beginners, have several sets with objects. Introduce students early to the tally mark method of designating "how many?". Tags may have a number on one side and objects on the other, and be kept in sorting boxes.

Tags groups of objects around the classroom. Purposely tag some groups incorrectly. Have children see whether they need to add or subtract some from the group to make the label correct. Ask questions such as: "How many more do we need?" "How many are left?" "How many shall we take away?"

Keep plenty of blank tags on hand. Paint Oilcloth Basic Work Form like this and use it first with objects and tallies, then with numbers:

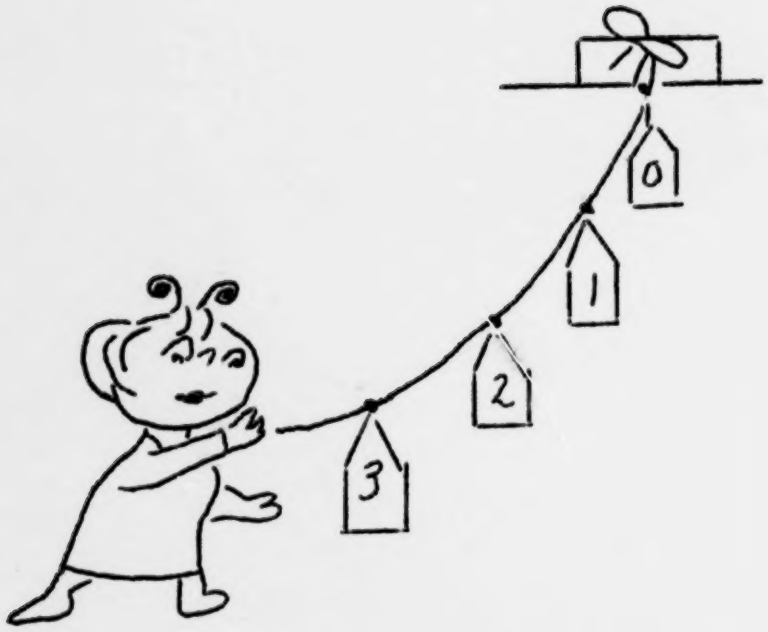


ALL KINDS OF NUMBER LINES

Actually, a number line is sort of a "working model" of our number system. It affords children an opportunity to discover, through personal experience, the structural principles which shape our system of quantitative thinking. When used as a laboratory device for the motor activities of young children, they discover its versatility and its importance.

Paint number lines on oilcloth, draw them on the floor, make them of clothesline, paint them on your playground, or hang them on the classroom wall. They wend their way right through the MATH WORKSHOP FOR CHILDREN.

One good way to show that a number line can go in any direction is to tie it to a brick or a building block. Ornament holders, clothespins, or large paper clips will hold tags which can be changed as you wish. Fasten one end of a cord which is several feet long around a brick. Tie knots at regular intervals to represent number stations. Zero to nine will probably be a good beginning, but later, jumps of two, three, four, five, and fractions may be substituted.



+	0	1	2	3	4
0	0				4
1		2		4	
2			4		
3		4		6	
4	4				8

-	8	8	8	8
2	6		6	
3		5		
4	4		4	
5				3

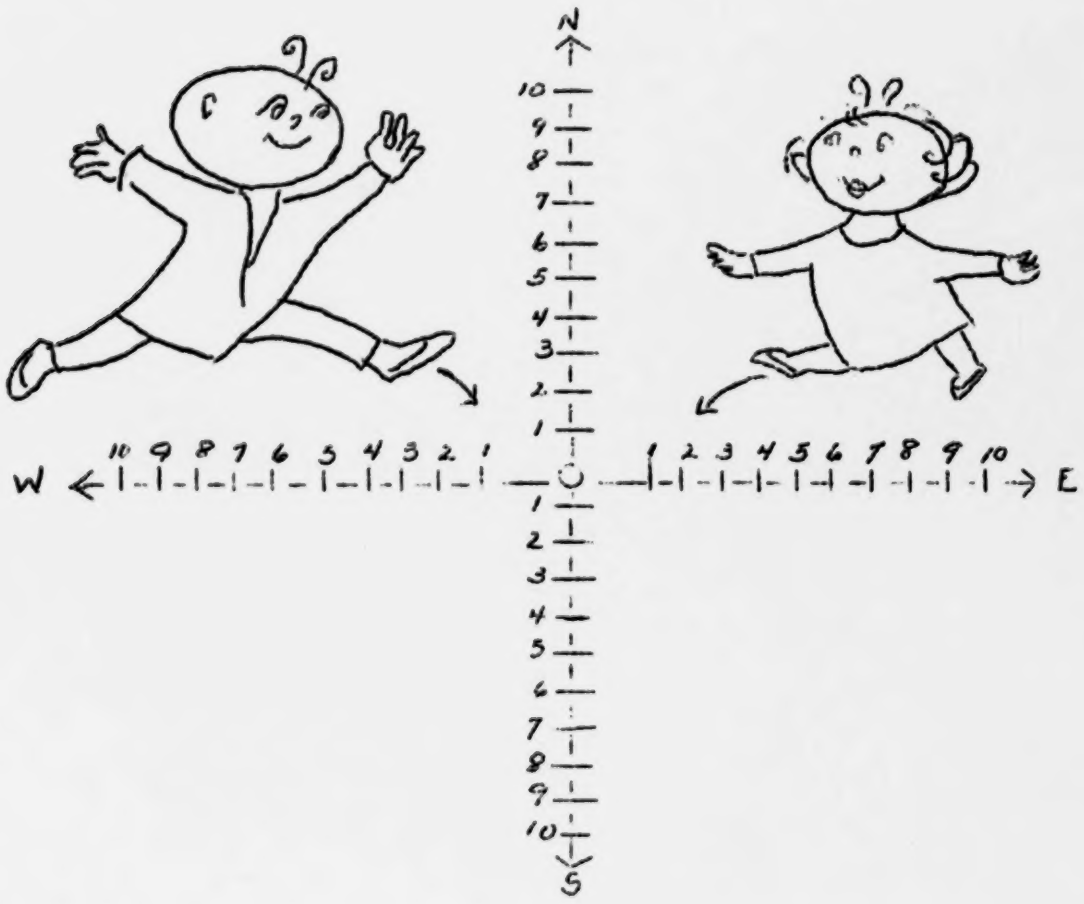
With this device, children observe that the number line can be moved to any angle above the brick if it is on the floor; and to any angle below the brick if it is raised. It is not always parallel; and we can skip around the use numbers in any direction, just as we can on the Playground Number Line Direction Finder described on the next page.

A PLAYGROUND NUMBER LINE

Mrs. Georgia Bergstrom, Principal of the Alta Leary School, Centennial District, devised a large Direction-Finder Number Line they can paint on their playground. A number line like this will supply a lot of mathematical thinking and learning on any nice day when the children can be outside. And just imagine an aerial view of Centennial!

This one will be pointed in the proper directions for north, south, east, and west, and painted with white paint - with zero at the crossroads. The stations from one to ten will have "landing places" large enough for youngster's feet, and the "size of the jump" will be about 16 inches. Numerals going in each direction, as shown, are advisable.

Using this kind of a number line, zero becomes another number and the numbers up or down, right or left, become opposite numbers. Young children can move about freely as they discover, establish, and reenforce important mathematical learnings.

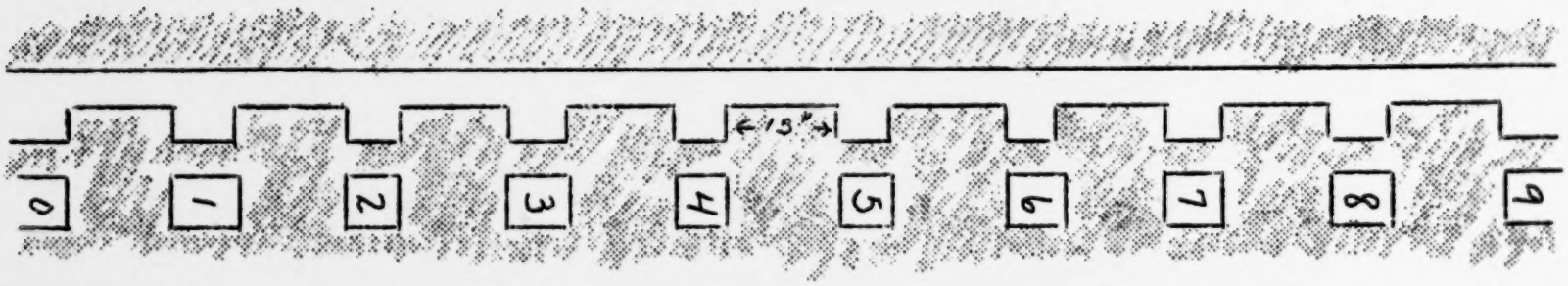


Some good activities might be: Starting at zero, jump 3 to the east ... 5 to the west ... now where are you? Start at ten north and jump 12 stations toward the south ... where are you now? Etc. Children could start at ten, one at each point - N, S, E, W. Using clapping or drum beats, directions could be given for moving toward the center. Who will get there first? How many more jumps must you make? Etc. Rhythm Instrument and Birthday number line activities described on the next pages could be conducted outside on this lively and appealing playground number line originated by Centennial.

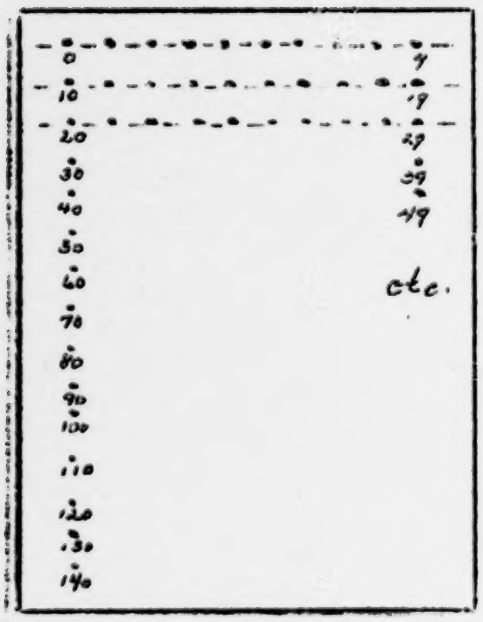
THE WALK-ON NUMBER LINE *

For introduction at Kindergarten, first or second grade, a number line drawn on the floor or hard-surfaced playground is ideal, and can be used for the motor activities of younger children.

It may be laid out on the classroom floor with masking tape or chalk, or permanently with paint. The 15" intervals suggested in this illustration are average; adjust them to suit the physical development of the pupils involved. While the line may be permanent, names of the points or stations should be printed on separate, removable cards. In earlier activities of jumping, hopping, skipping, walking, and counting, cards are not used. Later, removable cards increase the variety and complexity of number line activities.



Children can "take trips" on the number line in a variety of ways. They can take "round trips," "be a number," "add up to a number," or "land between numbers." This can be a dozen, a foot, and if you bend it in a circle clockwise, it becomes a clock.



A NUMBER LINE CHART *

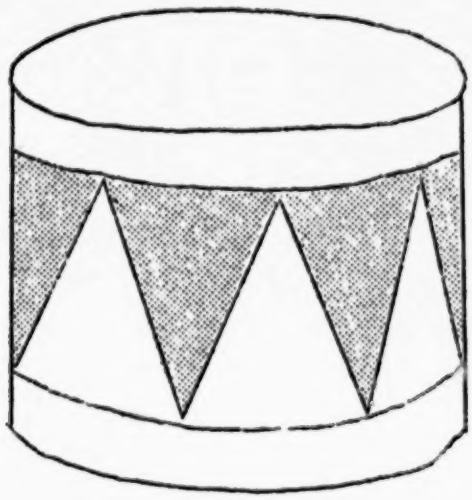
You can make a number line chart like Basic Work Form Nil.* Heavy, white cardboard, a magic marker, and your own interval measurements are all you need. Lines, dots, and numbers extend from 0 to 149.

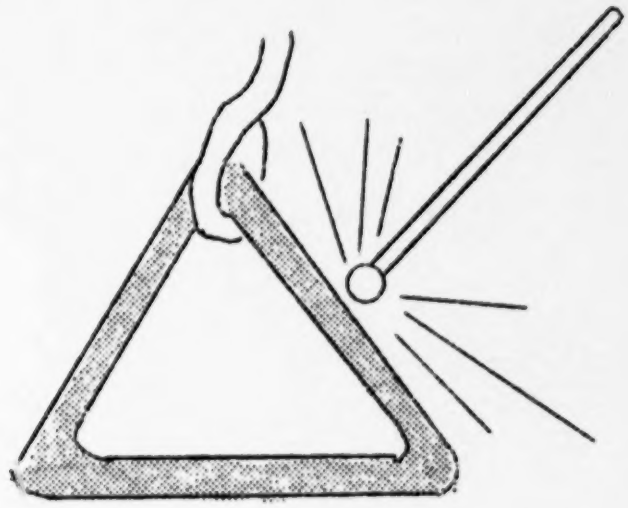
Its uses are many: "chunks" of it may be used. You can go forward or backward, up or down, across, over, and around to add, subtract, multiply or divide. Record keeping starts early and extends to many levels of complexity.

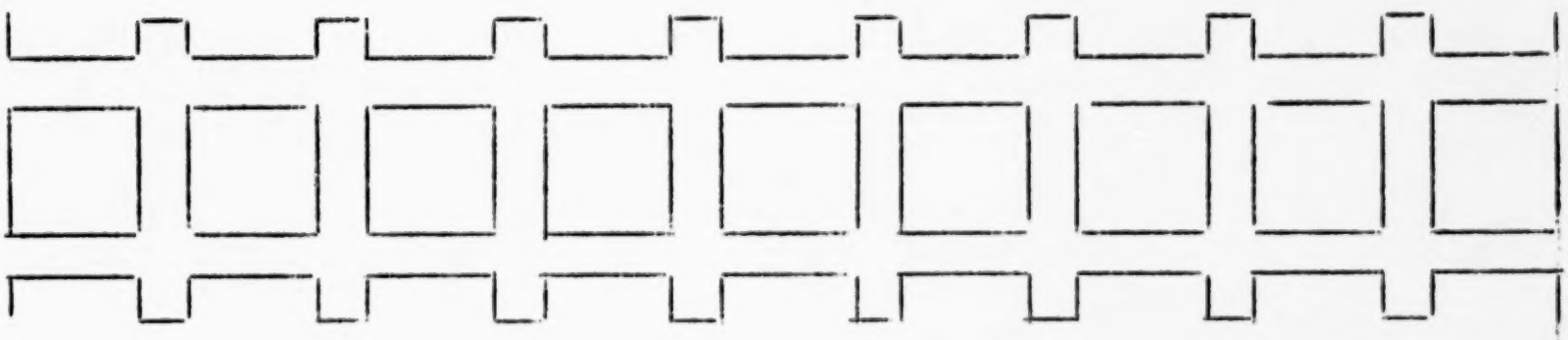
* Adapted from Wirtz, Botel MATH WORKSHOP FOR CHILDREN.

A RHYTHM NUMBER LINE

Tiles on the classroom floor, your classroom floor number line, or a portable folding or clothesline number line can become a railroad, or a street passing children's homes, or city blocks. Using number lines in this way, students get the idea that something has gone before and something will come after, no matter what particular "chunk" you are working with at the moment.







Use drums, sticks, triangle, clapping or piano to beat out jumps, trips, steps, hops, giant steps, etc. The teacher will, of course, do this at first - but children quickly take over the job of "beating out the number."

Another child can keep tally on the board or counting frame. Others keep a record on their desks with Sorting Box objects, tallies or numbers.

<u>Single number line suggestions</u>:

1 beat, 2 beats, 3 beats - How many in all? Where are you now?
Left 1 beat, right 2 beats - Where are you now? Etc.
Jump 2 to the right and 2 to the left - What happened?

<u>Double number line suggestions</u>:

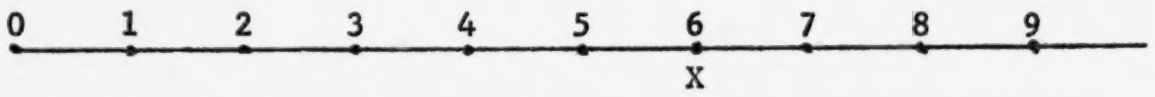
Use each side in same way, with a different rhythm instrument for each line. Combine scores of travelers, figure the difference in stations, etc. Later, try quarter-note, whole-note, and half-note beats, etc. Now drummers, travelers, and scorekeepers have a real challenge.

-- Sara Fox
Doylestown Township

-- Jacqueline McCauley
Manor School, Pennsbury

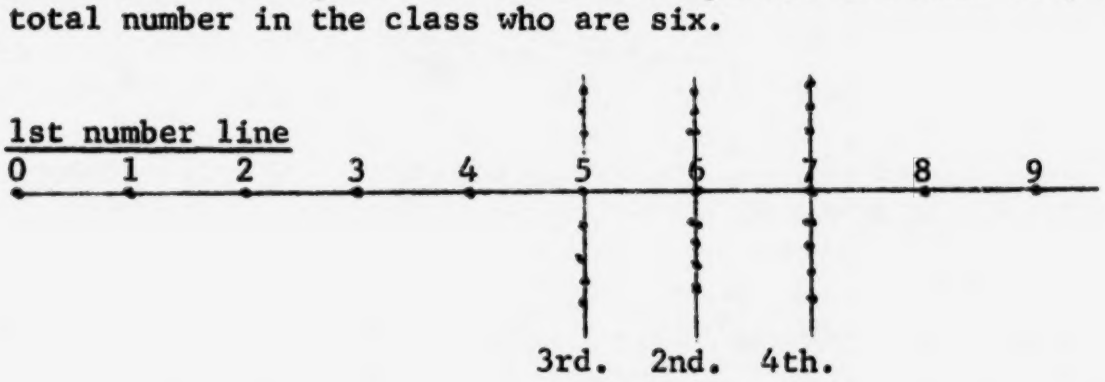
THE BIRTHDAY NUMBER LINE

An activity which is packed full of learning and can become a special event on class birthdays is a number line honoring the birthday child. Counting, grouping, adding, subtracting, and thinking about a new situation, an added year, are important activities for five- and six-year-olds. For this, you will need 4 number lines - gay plastic clothesline is good.



Lay one number line as above. Have birthday child count to new age and stand on that station (X). Ask several questions: How old are you today? How old were you yesterday? How old will you be next year?

Put a second number line across the first at the birthday child's station. Have the girls and the boys who are six stand on that line. Let the birthday child count the girls and then the boys who are also six, and tell the total number in the class who are six.



Now add a number line for those who are five and those who are seven. Continue to ask same kinds of questions: How old were you last year? How old next year? How old in one year, two years, three years? Make a chart like this:

	5 years	6 years	7 years
Boys			
Girls			
Total			

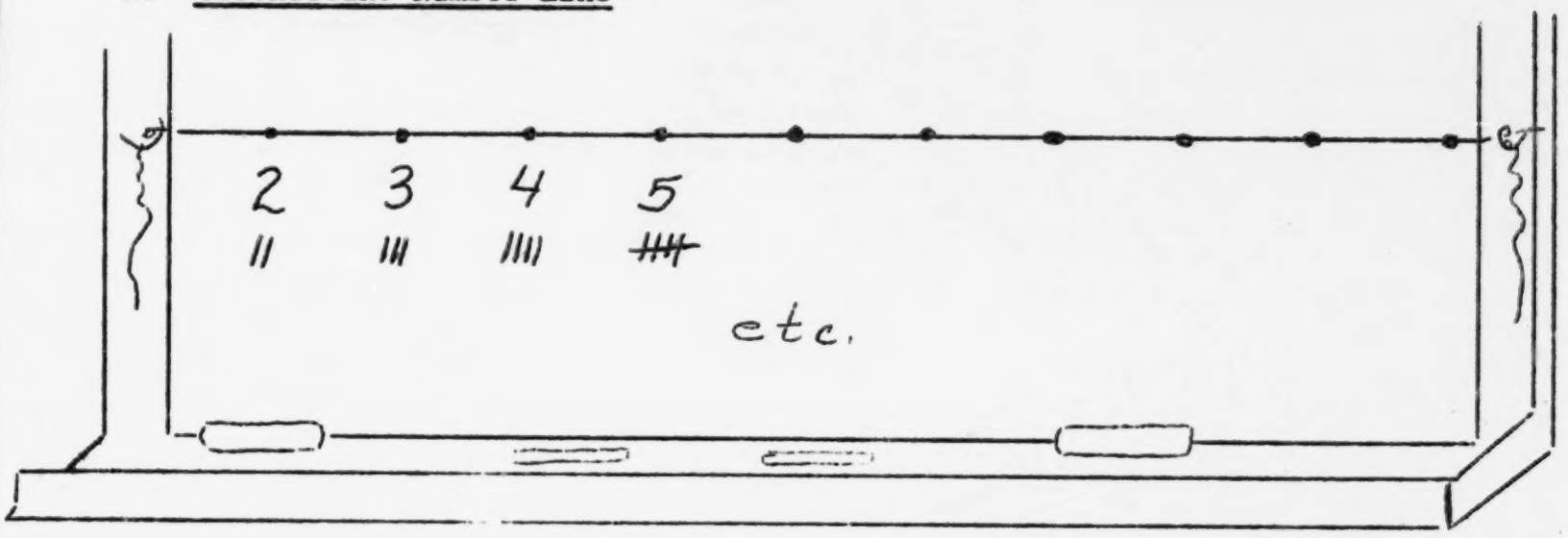
Class Total_____

Ask when these totals will be changed. Lead class to three conclusions: When someone else has a birthday, when someone moves, when we get a new pupil.

-- Katheryn Laning
Warrington Township

MORE NUMBER LINE IDEAS

A. Clothesline Number Line



Plastic clothesline makes a fine portable number line. Stations may be knots or marks made with nail polish. This number line can be stretched across the blackboard on two hooks, and placed or taken down quickly and easily. Use board for appropriate numbers or letters.

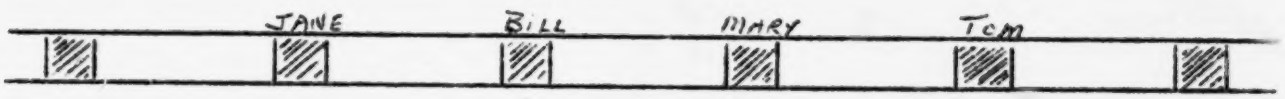
-- Jeanne Kryzwicki
Pennsbury

B. Folding Number Line

Cut uniform strips (15" - 18" length) of cardboard and join with mystic tape. Join end-to-end so that strips may be folded back over each other when not in use. Make tags from oak tag, punching a hole for opened paper clip hooks, and reenforce the holes with gummed notebook reenforcements. Hang numbers and letters on a coat hanger when not in use.

-- Marguerite Glenn
Warwick Township

C. Using a number line on the floor, have a child at each number station:



Who is third? first? fourth? second?
If John stands beside Tom, what will he be?
How far is Jane from Mary? from Tom? Etc.
To make a record, use this device:
To reverse procedure: In which place is Tom? Jane? Bill? Mary? John? Etc.

Place	Name
1st	Jane
2nd	?
3rd	Mary
4th	?
5th	?

-- Marion Tomlinson
Churchville

GROWING UP WITH THE NUMBER LINE

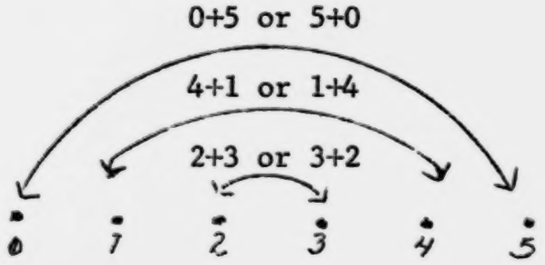
After your children have had many experiences with "How many names has a number?", and have used objects, tally marks, tags, and jumps to discover many interesting number facts, you might enjoy trying some more advanced activities. The number line continues to be a "working model" of our number system for many kinds of operations such as these:

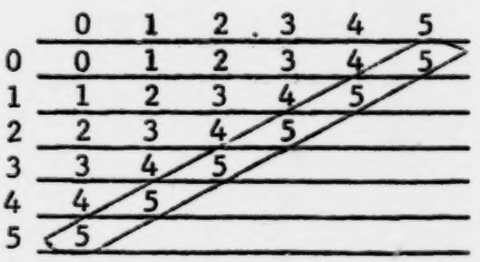
Use your floor number line, or a portable number line or ladder. Give children tags numbered from 0-10 and have them arrange themselves in sequence.

1. Starting with the combinations which make 5, ask questions such as these: Number 5 step forward. Can the person who has the number we add to this one to make 5 come forward, too? (That's right - 0.) Number 4 step out. Who has the number we add to 4 to make 5? Continue in the same way with 3, 2, 1, and 0.

2. A record of what you have just shown could look like this: or this: or this:

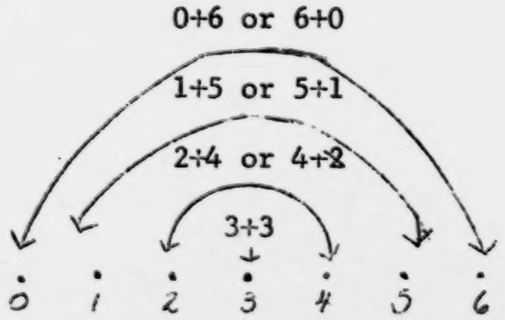
5+0 = 5
4+1 = 5
3+2 = 5
2+3 = 5
1+4 = 5
0+5 = 5

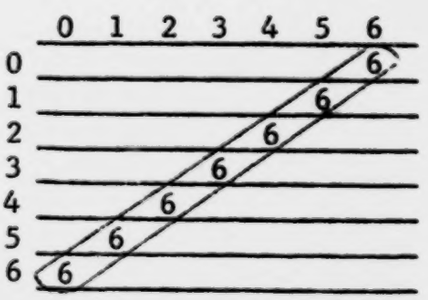




3. Now, let's try 6. How many ways can you find to make 6 using two numbers which total 6? (7)

6+0 = 6
5+1 = 6
4+2 = 6
3+3 = 6
2+4 = 6
1+5 = 6
0+6 = 6





4. Continue this same kind of questioning and action, letting students keep three kinds of records illustrated above. How many ways are there to make 7? Are you sure?

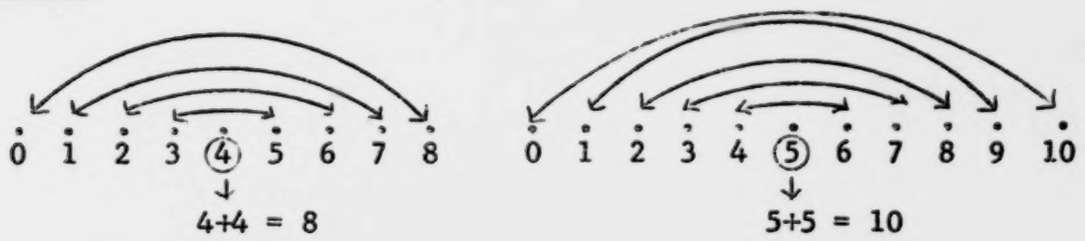
5. If we continue this same kind of questioning and action through the combinations which make 8, we might make some generalizations: Do you think you know for sure how to draw these kinds of pictures (or keep these kinds of records) of "what's happening?" Make a record for 9 and let's see if you're right. What about 10? Would it work for 16 or 25 or 80 or 100?

What happens when we write the possibilities like this?

0+7 = 7	8+0 = 8
1+6 = 7	7+1 = 8
2+5 = 7	6+2 = 8
3+4 = 7	5+3 = 8
4+3 = 7	4+4 = 8
5+2 = 7	3+5 = 8
6+1 = 7	2+6 = 8
7+0 = 7	1+7 = 8
	0+8 = 8

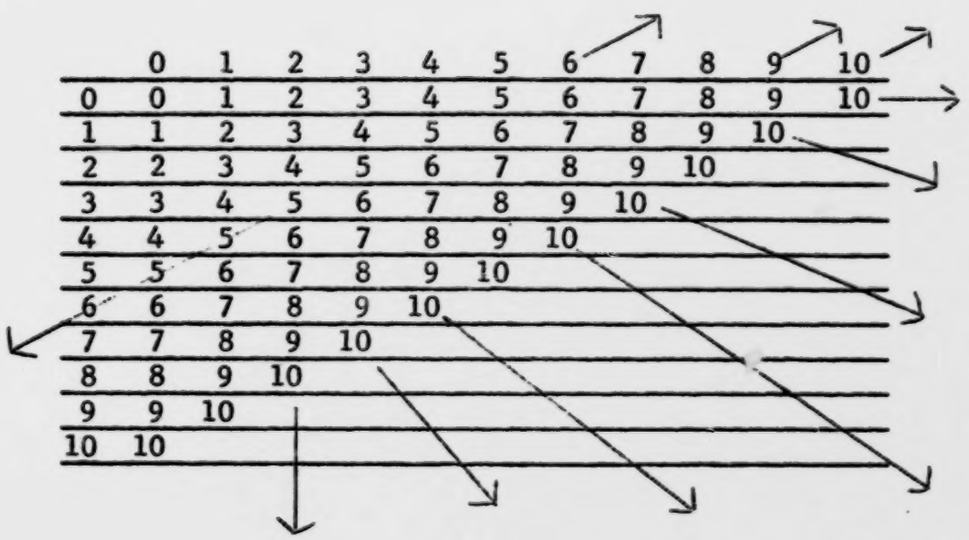
For 5, there are 6 ways
For 6, there are 7 ways
For 7, there are 8 ways
Etc.

Like this?



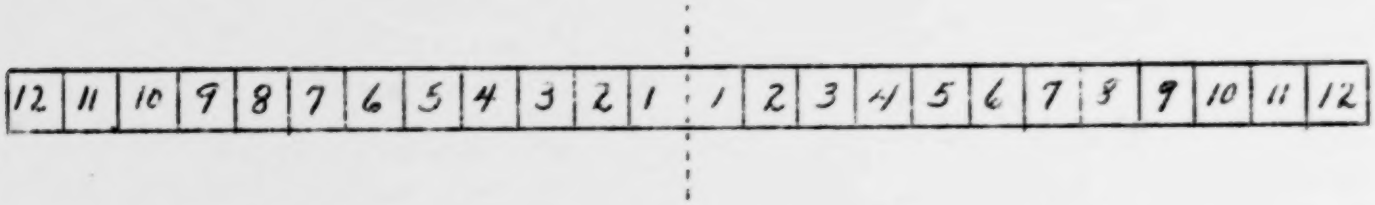
When we work with odd numbers, our two-way arrows come out even. When we work with an even sum, we have an odd number left over. Double this odd number, and it makes the original sum.

Like this?



Do you think they could go on and on in all directions? Could we keep building this table?

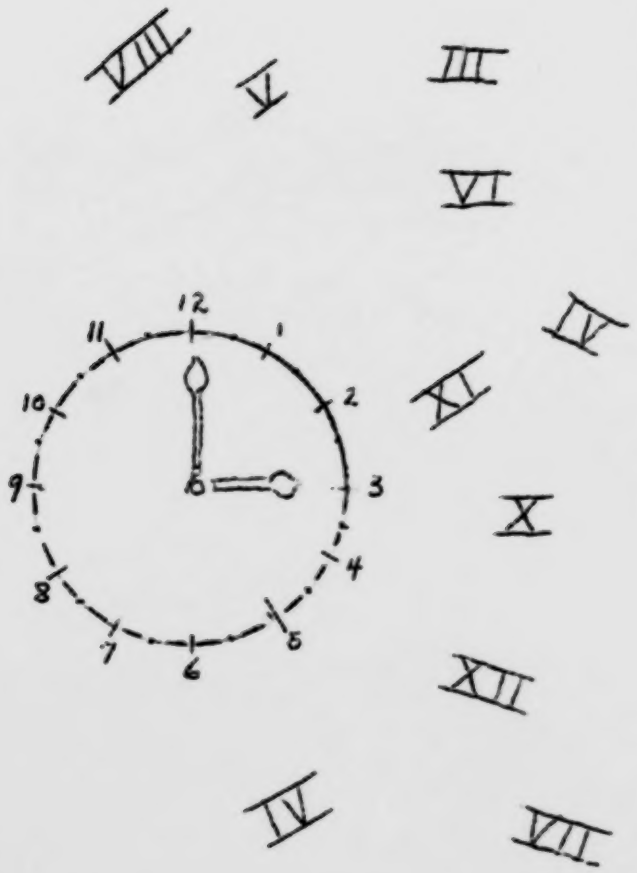
BACK TO THE NUMBER LINE



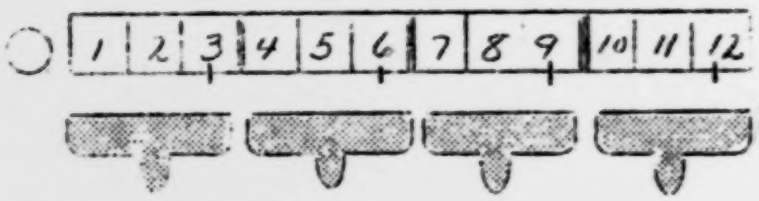
One oak tag strip numbered from one to twelve on each side and small enough to handle (3" x 36") can serve many purposes. When not in use, it can be rolled, secured with a rubber band, and kept in the MATH RESEARCH CENTER.

Small "paddles" in sizes 1, 2, 3 and 4, as described on page 27, can be used to add, subtract, multiply, and divide. Paddles numbered on one side and plain on the other have a variety of uses. This small set of paddles would be 3", 6", 9", and 12" x 3".

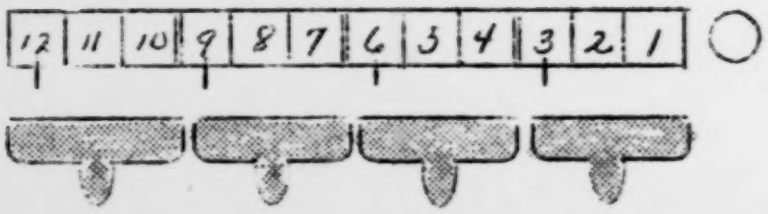
With the reverse side out, join the two ends and this number line becomes a clock, with the 12 at the top, the 6 at the bottom, and the 3 and 9 opposite each other. Experiences with the paddle will help children determine quickly where the half and quarter hours are.



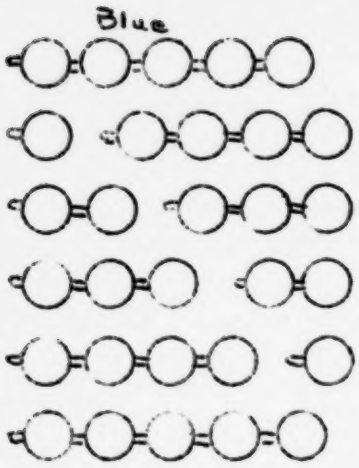
How many threes are there in twelve? Put a mark in the middle of the last numeral of each group of threes. (3, 6, 9, 12 - left to right.)



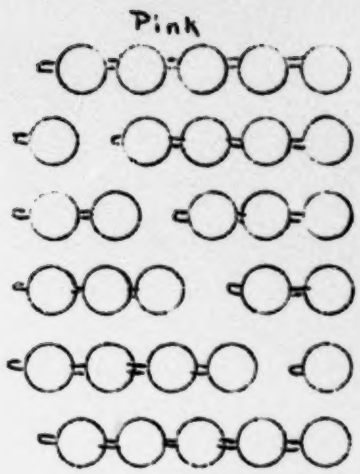
Can you reverse the number line and do the same thing going from right to left? Or from left to right?



With this number line and small 1" by 12" number lines which the children can manipulate themselves, you can introduce not only the clock, but the ruler, one dozen, seasons and months of the year.



POPPIT MATHEMATICS



If Poppits haven't gone out of style this year, have the girls take them off or ransack their mother's jewelry collection at home. Your students will have an adventure rich in mathematical possibilities.

5 + 0 = 5
1 + 4 = 5
2 + 3 = 5
3 + 2 = 5
4 + 1 = 5
0 + 5 = 5

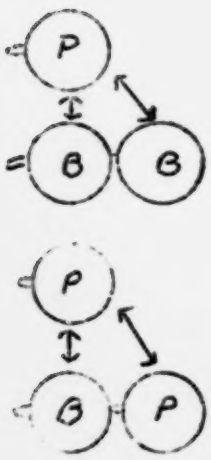
With one string of one color, students can explore such possibilities as "How many ways can you make three, four, five, six, etc." Another group might be doing the same thing with a second color. Have them keep a record of the ways they find to combine one color for the various combinations or "names for a number" in the form of one number plus another.

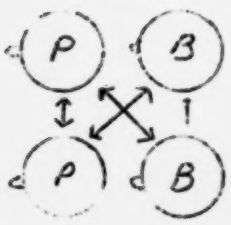
0 + 5 = 5
1 + 4 = 5
2 + 3 = 5
3 + 2 = 5
4 + 1 = 5
5 + 0 = 5

Poppits divide easily into connected groups. Questions such as: "Can you think of other ways?" and "Do you think you have found <u>all</u> of the possible ways?" stimulate good thinking.

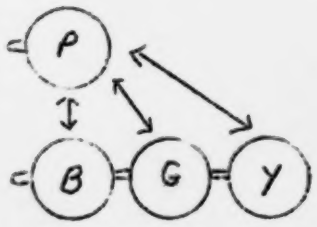
Now, suppose some students are interested to see how many ways they can make 5 with two colors? Have them write the various combinations they find, with a question such as: "How can we keep a record of the combinations we find?"

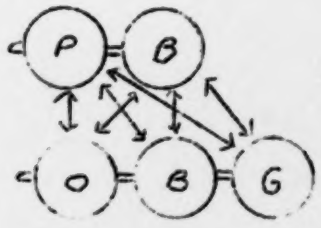
Ask the kinds of questions we could ask about making couples of boys and girls, or outfits of skirts and blouses (or shirts and pants). "If we have one blue bead and two pink beads, how many pairs can we make? 2 pink beads and 3 blue beads? 3 pink beads and 3 blue beads?" Etc. With a simple device such as Poppit beads, you can go directly into the heart of multiplication without ever giving it a name, and first graders love it! So do older students!





P + P
P + B
B + P
B + B





WALLPAPER ARITHMETIC - number strips *

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150

20 Yellow 1's

12 Lt. Green 2's

8 Lt. Red 3's

8 Med. Green 4's

4 Lt. Blue 5's

4 Med. Red 6's

2 Gray 7's

2 Dk. Green 8's

1 Dk. Red 9

1 Dk. Blue 10

A PICTURE IS WORTH 1,000 WORDS*

Wallpaper Arithmetic is a fascinating name for a gascinating game. The number chart can be made from white cardboard, 15" x 22½". It is interesting to make one chart with numerals from 1 - 150 completely filled, and one with the first fifty numerals left blank.

The blocks on the chart are then 1½" x 1½" square. The strips start with yellow ones which are 1½" x 1½", light green twos which are 1½" x 3", light red threes 1½" x 4½", etc.

Note that one is yellow; two, four and eight are light, medium, and dark green; three, six, and nine are light, medium and dark red; seven is gray; five and ten are light and dark blue.

---

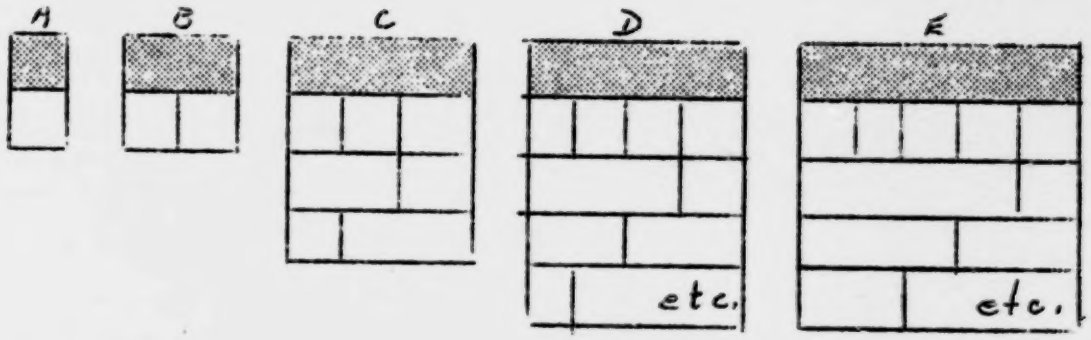
* Adapted from Wirtz, Botel Number Boards and Strips.

How to use Number Strips and Boards:

At first, encourage the child to make up simple patterns of trains, walls, ladders, castles, or even multi-stage rockets. When he has given his picture a name, ask him to make two or three more with the same number of pieces as his picture.

Suggest making three "trains" the same length and the same number of pieces.

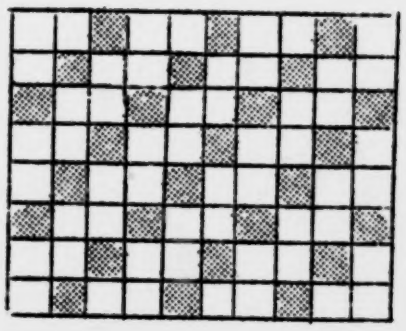
With each number strip, how many different "trains" can be made the same length but using different pieces?



Number strips are r.. numbered, and although they may represent 1, 2, 3, 4, 5, etc., they also suggest anything we wish them to just by "changing the rule."

The primary purpose of using color for the number strips is to facilitate talking about them. Light blue can then be ½ or whatever we choose to make it. What would the others then be?

Revealing patterns jump out quickly as the cells are covered by the unit squares according to certain simple rules such as:



A. Cover every third cell (all multiples of three).

B. ..... every even-numbered cell.

C. ..... every seventh cell ... every eleventh cell.

D. Multiples of three and four superimposed on the same board. (3x4 = 12; consequently, we would cover 12, 24, 36, 48, etc.)

E. Multiples of nine and eleven ..... Will the same kind of pattern develop with multiples of any 2 numbers?

As children use these strips, they can get into more difficult addition, subtraction, multiplication, and division. They develop a healthy feeling of self-confidence when they tackle harder problems with tools they already have and can use. Record keeping (number of pieces and patterns) should be encouraged.

PAPER DOLL ARITHMETIC

Another version of "Poppit Arithmetic" can be developed with paper dolls, believe it or not. Multiplication really asks: "How many ways can we pair the elements of one group with the elements of another group?"

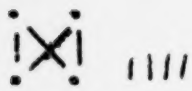
The problem, in its simplest form, would ask:

> <u>If you are a girl</u>, how many outfits can you make with one skirt and no blouses? one blouse and no skirts? one skirt and one blue? with one skirt and two blouses? with two skirts and two blouses? with three blouses and one skirt, two skirts, or three skirts? Etc.
>
> <u>If you are a boy</u>, how many different outfits can you make with one shirt and one pair of pants? one pair of pants and two shirts? three shirts? two pair of pants and three shirts? Etc. <u>Count the different outfits</u>.

<u>Problem example</u>:

Mary: I have 6 outfits.
Jane: I have 9 outfits.
Mary: I have 3 skirts.
Jane: So have I.

How many blouses has Mary?
How many blouses has Jane?
How many blouses together?



Have cut-out girl for the girls and boy for the boys. Make blouses and skirts and shirts and pants in two colors at first, then add a third color to each part of each set, etc.

Young children can dress dolls in different outfits and while they are doing this we can keep a tally record of combinations.

Older children can visualize the various combinations and work them out on more complicated tables. A rich mathematical seed will be implanted that will grow and <u>grow</u> and <u>grow</u> through the rich, brown earth that <u>is</u> real mathematics.

Blouses

Skirts	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

Shirts

Pants	1	2	3	4	5
1					
2					
3					
4					
5					

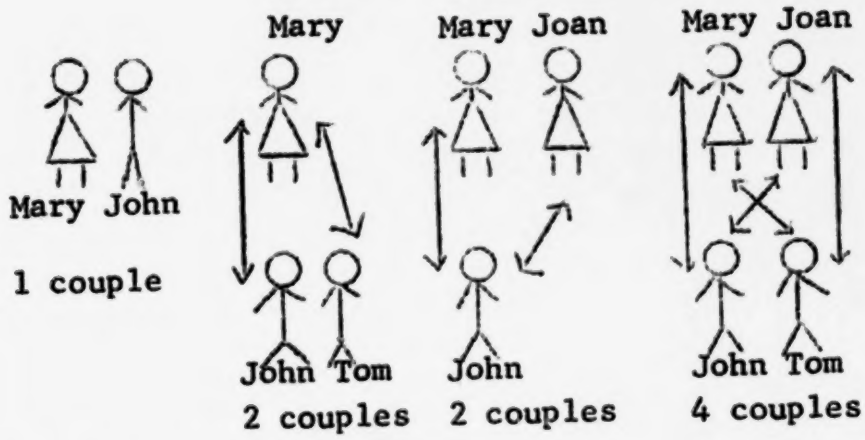
Same Record

What's happening here?

REAL LIVE COUPLES

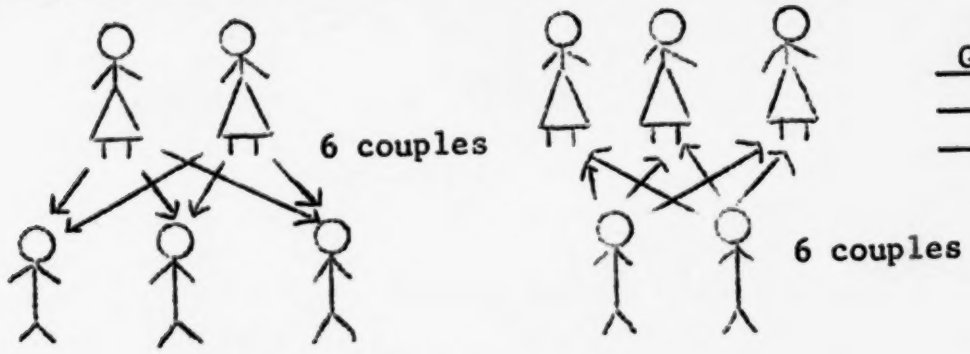
In the classroom, we might start with the agreement that a pair, or "couple," must be a boy and a girl.

Ask a girl, say Mary, to come to the front of the room. Is Mary a "couple?" (No) Ask John to come up, too. Now can we make a couple? (Yes) Ask two more girls and two more boys to come forward. We now have 3 girls (Mary, Joan, and Sue) and 3 boys (John, Tom, and Jim). How many couples can we make with 2 girls (Mary and Joan) and 1 boy (John)? How many couples using 2 girls (Mary and Joan) and 2 boys (John and Tom)? Let's see:



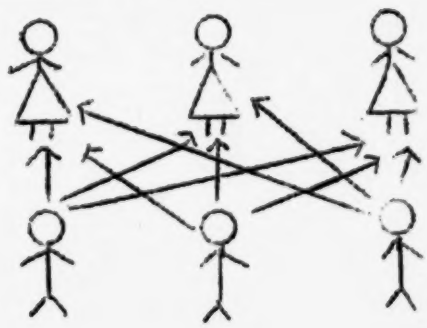
Girls	Boys	Couples
1	0	0
0	1	0
1	1	1
2	1	2
1	2	2
2	2	4
3	2	?

What about 2 girls and 3 boys? or 3 girls and 2 boys?



Girls	Boys	Couples
2	3	6
3	2	6

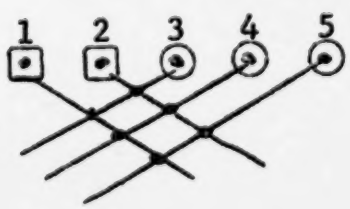
Now how many couples can we make with 3 boys and 3 girls?



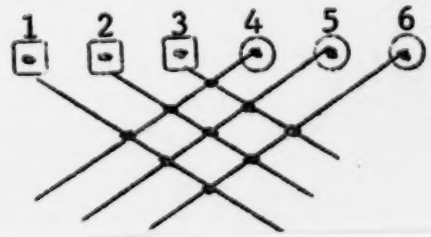
Girls	Boys	Couples
1	0	0
0	1	0
1	1	1
2	1	2
1	2	2
2	2	4
2	3	6
3	2	6
3	3	9

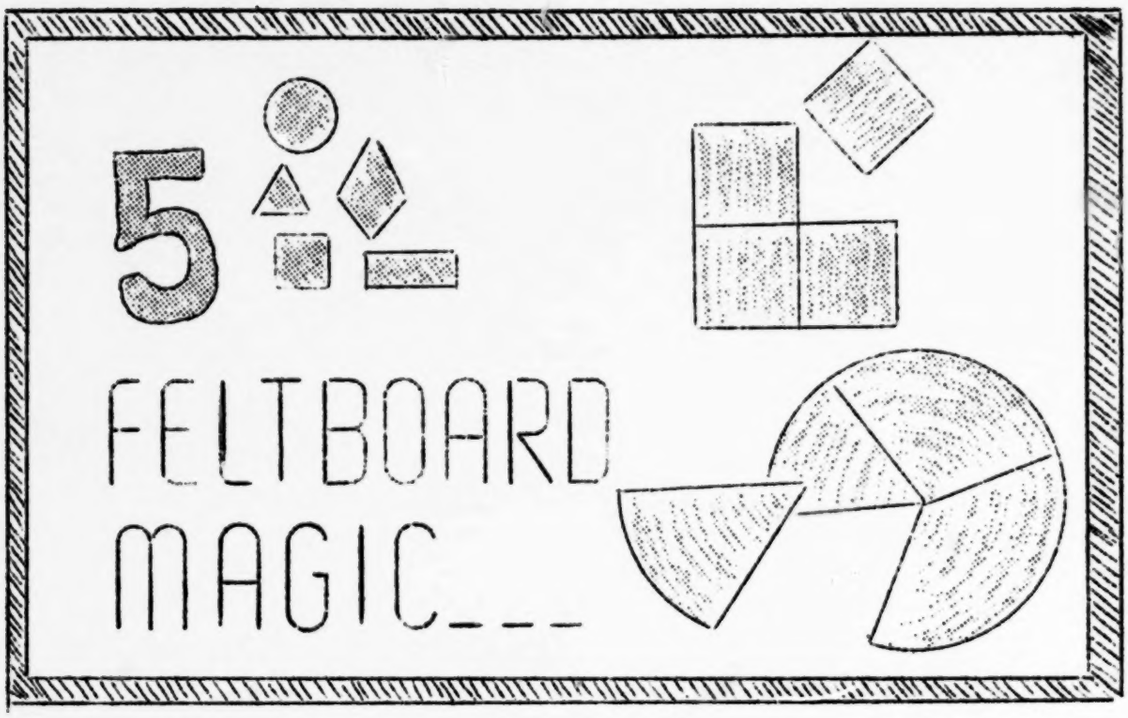
Obviously, if we actually let children "act out" experiences like these long before we introduce formal multiplication, we can give them a very clear picture of what we mean later when we say: 3x2 = 6 and 3x3 = 9. Children can discover the true meaning of multiplication, as different from addition, every time we use an illustration to show that:

2+3 = 5 but 2x3 = 6



3+3 = 6 but 3x3 = 9





The feltboard is a valuable piece of classroom equipment and adds interest and variety to almost any subject. It lends itself to a variety of materials, shapes, colors, patterns, and manipulations which are mathematically rich!

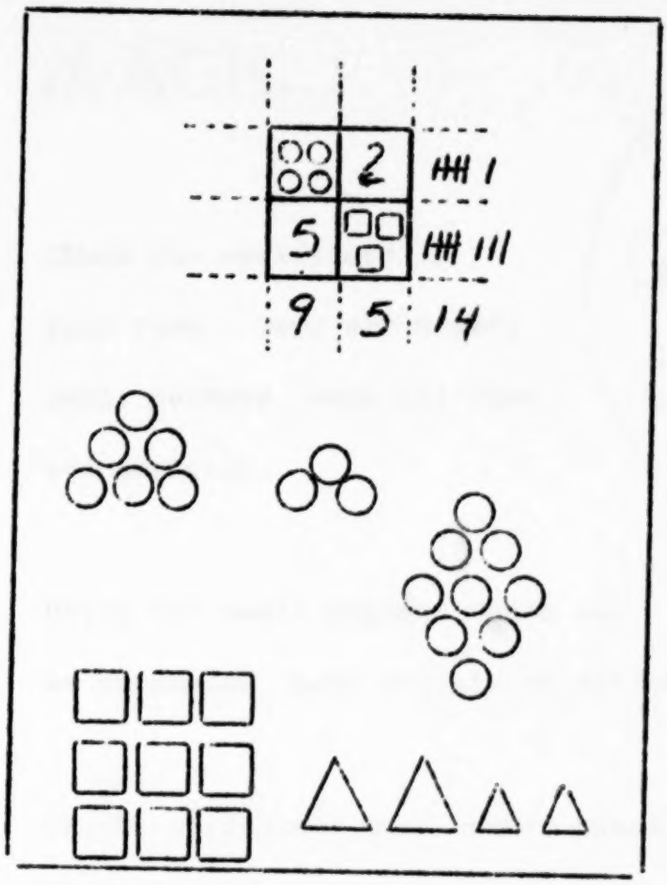
You can make a feltboard easily. Use heavy cardboard, wallboard, or plywood - 18" x 24". Stretch lightcolored flannel or felt (felt is best) over it, folding the edges over and gluing or lacing them. A compartment on the back holds materials.

The apparent magic of a feltboard is due to the instant adherence of other materials to its surface, including another piece of felt or flannel. Materials with good clinging capacity include sandpaper, suede paper, lightweight wools and yarns, and many other lightweight, rough-surfaced substances.

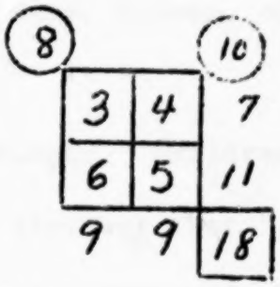
Suggestions for using:

Make shapes and numerals for matching, adding, subtracting, and observing.

Cut squares, circles, rectangles and triangles into halves, thirds, fourths, etc. By "finding the piece that's missing" or "making the rest of the picture," your students will be multiplying, dividing, and doing "fractions" long before they're ready for the names of these number functions.



MAGNETIC MAGIC

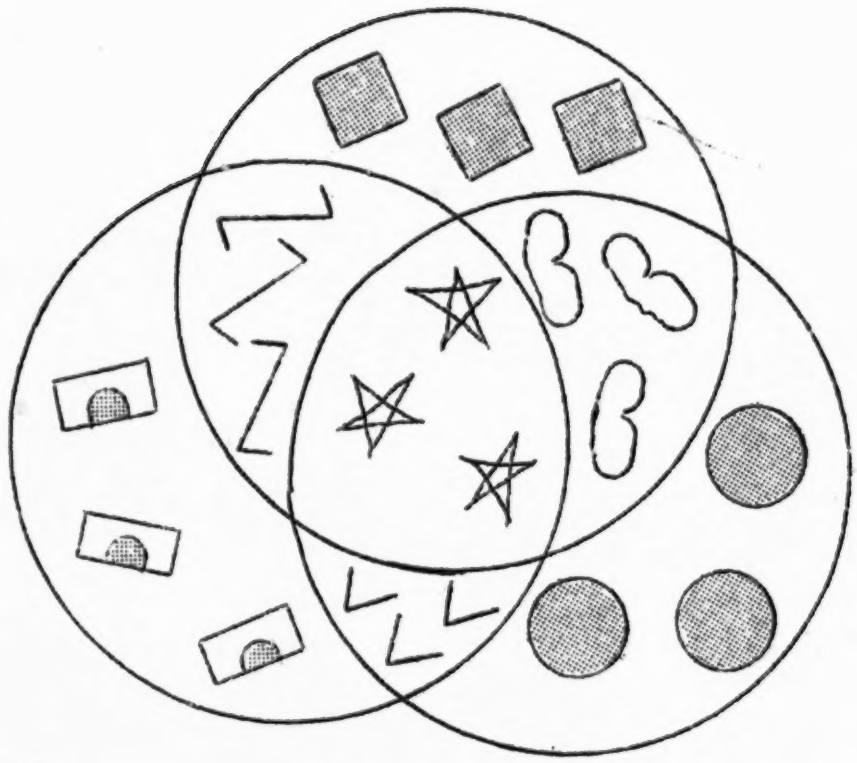


Use a commercially made steel board or paint an 18" x 24" sheet metal rectangle with any dark, flat paint. Be sure there are no sharp edges or corners - they must be rolled back. You can buy magnetized objects or attach small magnets to objects made of cardboard, plastic, or lightweight metal. (Use household cement for this.)

On the top half, paint this Basic Work Form with white paint. This particular form is adaptable to many number manipulations. Use numerals, objects, geometric forms, etc. Add, subtract, multiply, divide - up, down, across, and diagonally. Will it work, no matter what numbers we put in?

The bottom half is free for drawing chalk basic forms which can be changed. It can also be used in the same way the feltboard on the previous page was used, for manipulation of objects, comparisons, and for seeing and using relationships.

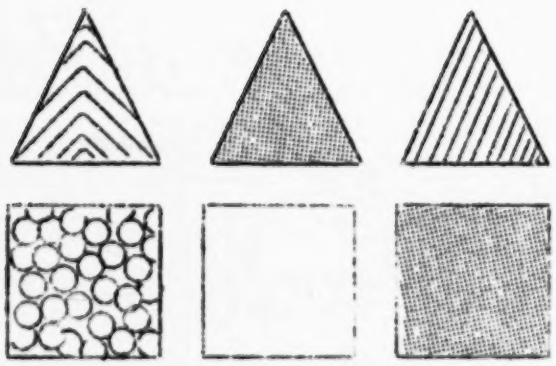
BLACKBOARD MAGIC



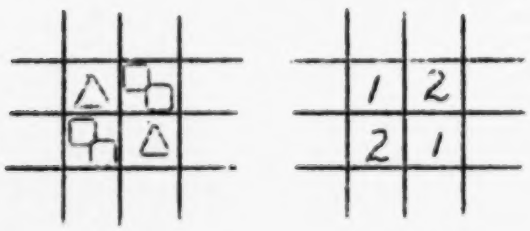
Check the chalkboard in your room. Many new boards (and, perhaps, some old ones) are magnetic.

Using the small magnets which may be purchased, make objects of various sizes, colors, and shapes.

On the chalkboard make number puzzle squares. Children may use these different shapes, colors, and sizes to discover that "numbers have many names," many colors, many shapes, and many sizes. Two red triangles and one square make three, just as two blue squares and one red triangle do.



WHAT MAKES 3?



<u>Do you see three things alike?</u>

<u>Can you make a group of three objects not alike?</u>

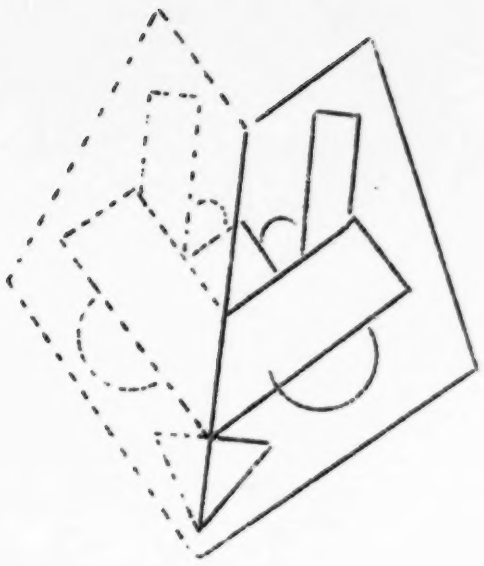
<u>How many objects are the same shape, but a different size?</u>

-- Grace Swaim
Churchville

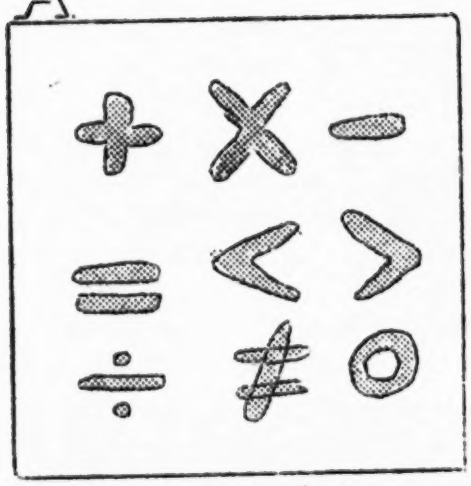
IMAGERY AND SYMMETRY

with

MIRRORS AND MOBILES



A.

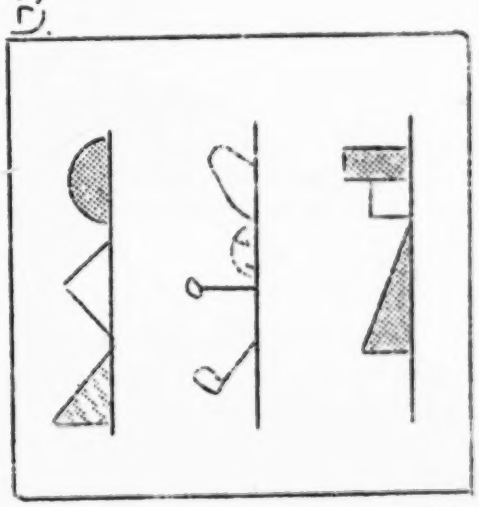


Roll clay in long ropes, break off pieces to reproduce these shapes.

Clay

Balance, symmetry, and proportion are important parts of the mathematical structure. Children learn size, color, shape, and number as well as discimination for balance or the lack of it through the use of manipulative materials made for seeing and feeling similarities and differences. Later on, the symbols which stand for groups of words also stand for big mathematical ideas: $>$ (more than); $<$ (less than); = (equal); $\neq$ (not equal).

B.

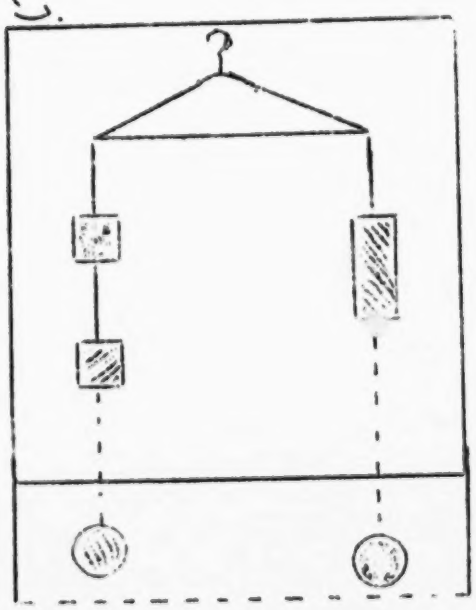


What is the other half?

Mirrors

Using a purse-size mirror, let pupils see the other half of pictures, such as these. (Encourage children to bring mirrors from home.) Demonstrate at the board, then let children try it with sorting box materials. Let them try to draw the "other half of a picture," or "fill in the missing part."

C.



Mobiles

Demonstrate a simple balance scale by hanging two like objects, one on each end of a coat hanger. Then show two which do not balance. Lead the class to discover why not. Make mobiles using doweling, coat hangers, reed, or balloon sticks for balance rods. The objects or shapes used are glued or scotch-taped to pre-cut lengths of string which are tied to the balance rod. Give each child two yellow squares and one strip twice as long. How are they alike? Will they balance? What else might balance a mobile? Commercial number "balance scales" may be used in conjunction with objects balanced on a clothes hanger.

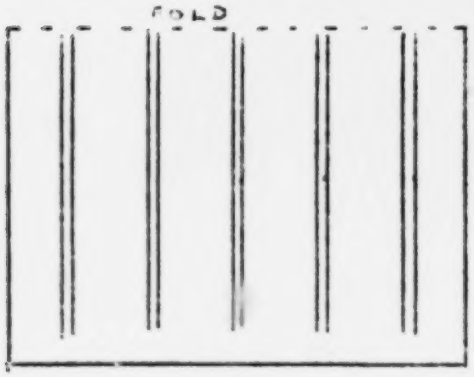
WEAVING TO TEACH.....
ROWS, PATTERNS, COLUMNS

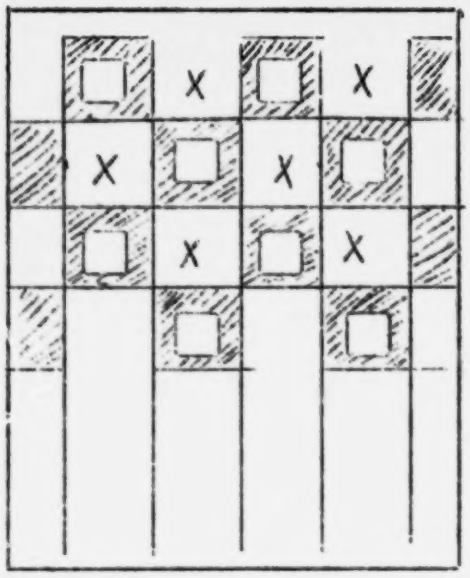
The search for patterns and relationships is of fundamental importance in mathematics. It is essential that we help students think in terms of pattern right from the beginning. Practically every page in the MATH WORKSHOP FOR CHILDREN has patterns for students to discover and develop.

Weaving provides practice in an activity which pays rich dividends - for children learn about rows and columns, over and under, across and down, left and right, horizontal, perpendicular, and diagonal. Each of these abilities will help them with recording, making charts and graphs, score keeping, predicting trends, and studying information compiled by others.

If you have looms and loopers in your classroom, they can be utilized as well as a simple loom with yarn. Knitting with large knitting needles fascinates little girls. Encourage them to make up patterns, other than the simple one suggested here.

For paper weaving, you could use:
1 sheet of blue, 9" x 12", paper per child.
Fold in middle and cut 5 slits, beginning ½" from edge and 2" apart.
Cut to within ½" of the end. Cut yellow strips 2½" x 9".





Place the folded blue sheet flat, and weave with yellow strips. Over, under ... Under, over ... Paste the end of each strip to the frame.

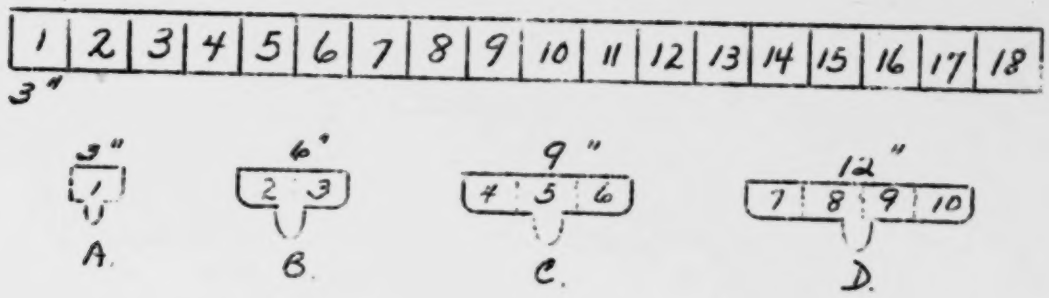
Good questions might be: How do you think the next row will start? How many columns in the mat? How many rows? What is the pattern? If we put an "x" on each yellow row, what could we put on each blue block to make an interesting pattern?

Try putting some patterns like these on the board and ask whether someone thinks he can continue them:

x o, x o, _____
oxo, oxo, _____
123, 123, _____
1, 22, 333, _____
$$, ## $$, _____

2, 4, 6, _____
1, 3, 5, _____
3, 6, 9, _____
1, 2, 4, 6, _____
(((,))),(((, _____

PADDLE-BOARD ARITHMETIC



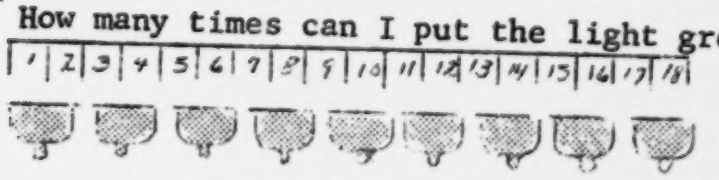
This is simply an oak tag strip, 3" x 64". It is divided into three-inch squares and numbered from one to eighteen. The paddles are 3", 6", 9", and 12" x 3", and should be made of heavy cardboard or even plywood. They are numbered on one side as shown above. The other side is left blank. Colors: A - yellow; B - light green; C - light red; D - medium green.

With these simple devices we have another approach to addition, subtraction, multiplication, and division. For young children objects or tallies could be placed on the paddles, or objects on one side, and tallies on the other.

Combinations of paddles held against the number line quickly reveal combination facts: If I hold the yellow and the light green paddle, what can I tell about 1 and 2? Light green and light red? How many? How many ways can we make 1, 2, 3, 4, 5, etc.? If we cover up one, how many are left? Cover two, three, four, etc. ... how many are left? Does it make any difference whether I cover 7, 8, 9 and 10 or 1, 2, 3 and 4 or 5, 6, 7, 8, etc., with the green paddle? How many are left? (All combinations to 9+9.)

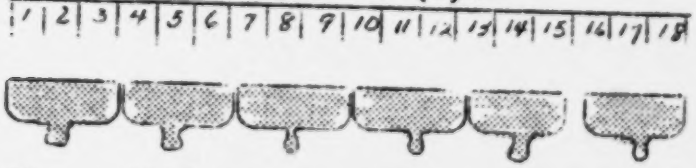
Multiples of numbers appear when we ask such questions as:

"How many times can I put the light green paddle (2) on the number line?"



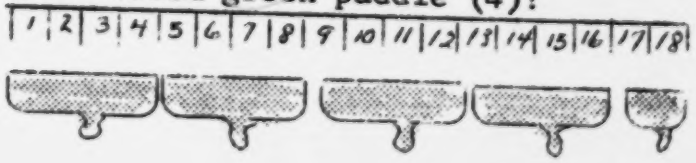
Exactly 9 times. What does this tell us about 9 twos? 2 nines? (18-4, 16-8, 12-4, 8-4, etc.?)

The light red paddle (3)?



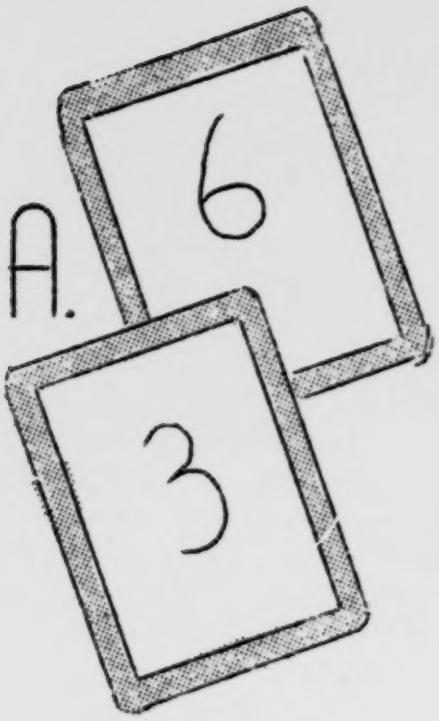
Exactly 6 times. Now what can we say about 2 threes, 3 threes, four threes, etc.? (3+3, +3, +3, etc.?)

The medium green paddle (4)?

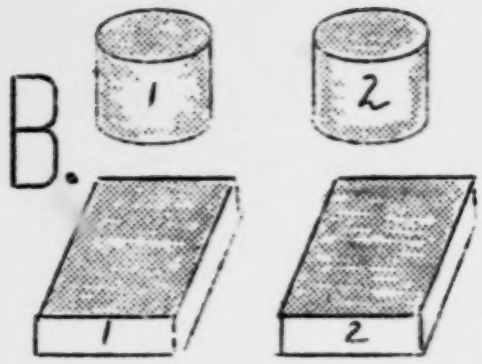


Four times and two left over. What can we say about 1 four, 2 fours, etc? (16-8, 8+4, 18-6, etc.?)

I'M THINKING OF A NUMBER .....



A.
Divide the class in half, each with a leader, and give each leader a regular or a magic slate. Each group decides on a number and gives the class one clue; that is, "under 10", "under 15," etc. - and the leader writes it on the slate. The other group tries to guess it in the fewest number of questions. 1 2 3 4 5 6 7 8. The best questions cut the problem down quickly: Is it one of these 4? Is it one of these 2? Is it 6? Then, you're thinking of 5.



B.
Put an equal number of objects in each box or container, starting with 5 or 6. Take some objects from one box in your hand. See who can guess how many you have by using the "fewest questions," as above. Now, how many are there in the box on your left? On your right?

Gradually increase the number of objects in each container and, taking some objects in each hand, ask these kinds of questions: How many in my right hand? How many in my left hand? How many in both hands? How many in box number one? box number two? How many are there now in both boxes?

C. What's My Rule?

I'm thinking of the first two numbers in a way that suggests the third one. What's my rule? When you think you know, give me an example - but don't give away the secret.

C.

4, 2	2
5, 3	2
6, 2	?
4, 1	?
5, 4	
7, 3	
7, 4	

4, 2	6
5, 3	8
4, 3	?
6, 3	?
6, 2	?
7, 1	
7, 2	

4, 2	4
5, 3	6
4, 3	6
4, 4	?
3, 5	?
7, 2	
8, 3	

4, 2	8
5, 3	10
3, 2	6
4, 3	?
5, 5	?
7, 3	
3, 2	

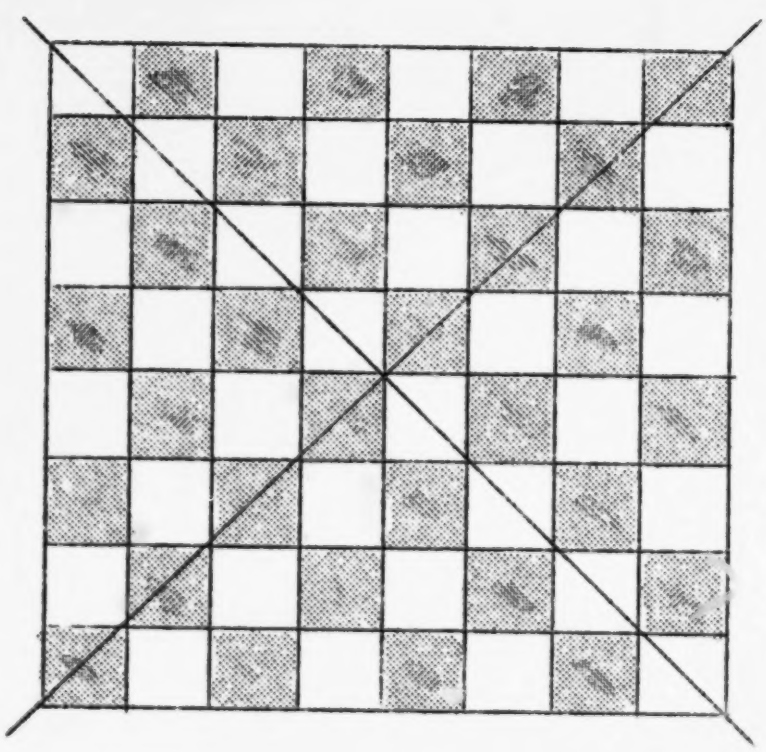
2, 4	3
3, 5	4
4, 8	6
2, 8	?
6, 10	?
2, 10	
5, 10	

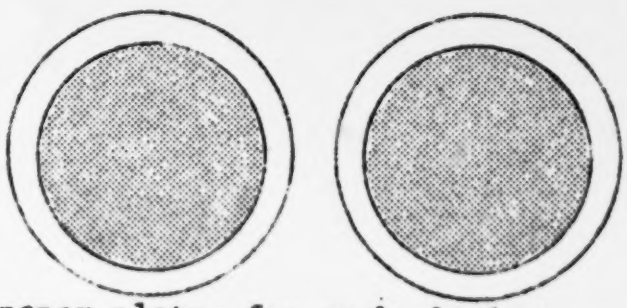
CHECKERBOARD ARITHMETIC

If your classroom has square tiles, you can make it into a huge checkerboard. Otherwise, experiment with taped or chalked squares. Later on, you can create more complex problems with a regular checkerboard or the miniature ones which can be purchased at the five-and-ten.

All checkerboards have the same number of squares and are arranged in the same pattern. Using a checkerboard as a model, ask students questions such as these: Do we have enough room to make a big checkerboard using our floor tiles? How many do we need on each side? If each tile is nine inches square, how long will each side of our checkerboard be? If we use two foil (or paper) plates stapled together to make each man, how many plates will we need to make our men? How many to make the black checkers? Red (or white) checkers?

Checkers can be numbered to make an adding machine in the lower grades and a multiplying machine in the upper grades. Children can play regular checkers on rainy days, using the plain side of paper plate checkers. If your floor has tiles of alternating colors, so much the better.

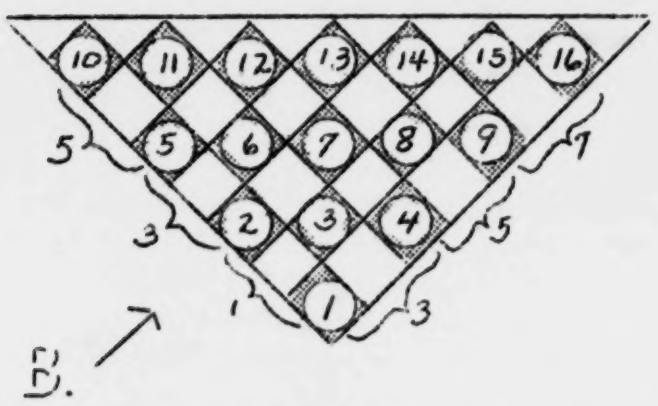




Two paper plates for each checker. Staple edges together & glue smaller circle of red or black to each side.

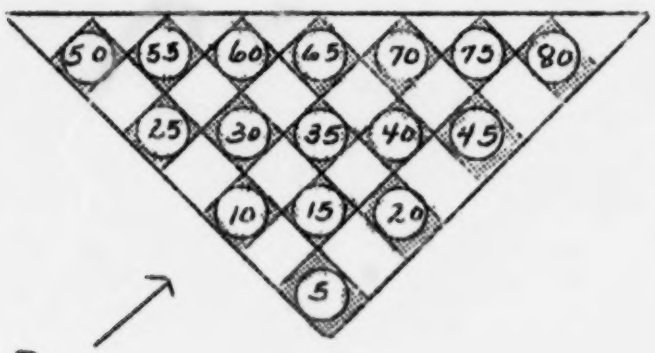
Looking at the floor checkerboard or a regular checkerboard <u>from any corner</u>, the squares give you an excellent <u>visual</u> record of odd and even numbers. Try it from any corner to the middle diagonal: <u>Row 1, one; Row 2, two; Row 3, three; Row 4, four, etc.</u> Count them. Children will discover that although there are eight blocks on each side, the corner blocks are shared by two sides, so there are 64 in all. <u>If we cut a board in half diagonally, how many whole squares? Horizontally?</u>

Cut commercially-made paper checkerboards into halves, diagonally. Checkerboards could also be duplicated. Each student will need 28 discs, numbered from 1-28, with extra blank discs to use as they think of "more things to try."



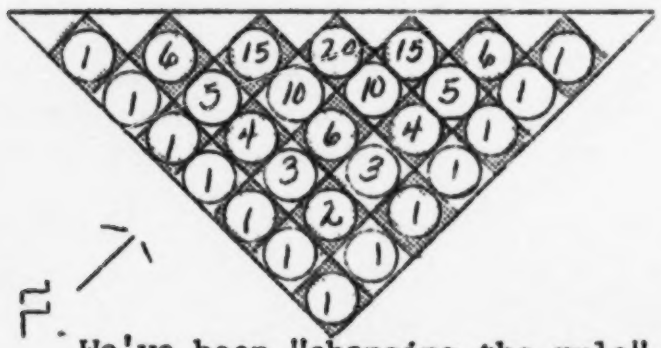
B.

Now try this. Note jumps are now odd numerals, even though size of jump was the same.



D.

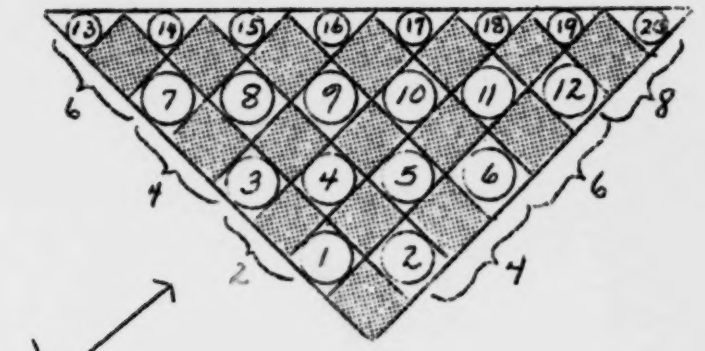
Try multiples of 3, 4, 5 or 6 on alternating rows. What pattern emerges on each of the three sides of triangles D and E?



F.

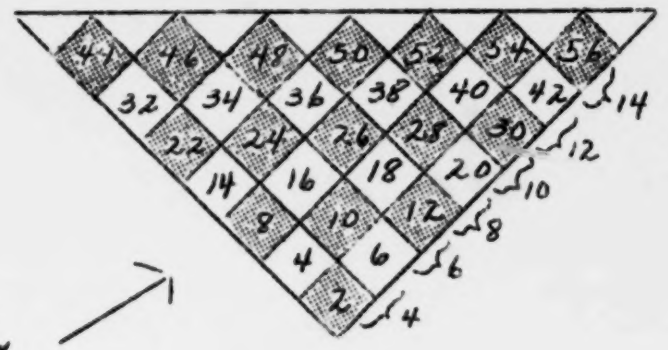
We've been "changing the rule" to make many interesting patterns and discoveries. Let's try putting ones along two sides. Now change the rule to say that each block must be the sum of the two numbers below it; working from the corner up, we can make Pascal's Triangle.

more checkerboard arithmetic ...



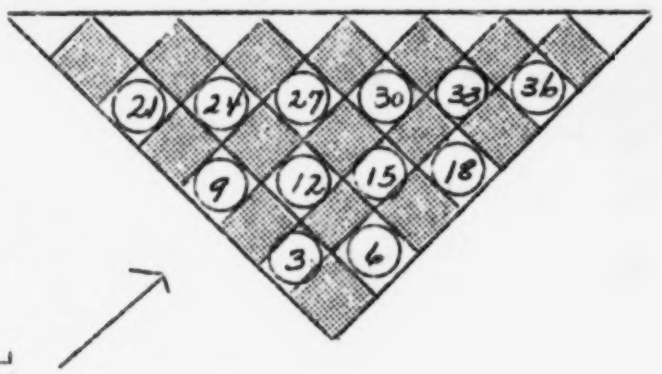
A.

Using counters, 1 to 20, put them on the even rows. Lead children to see jumps of one on diagonal and jumps of 2, 4, 6 and 4, 6, 8 on sides. Would this continue?



C.

Try putting all odd numbers or all even numbers on each row rather than alternating rows. What are the jumps on all sides now?

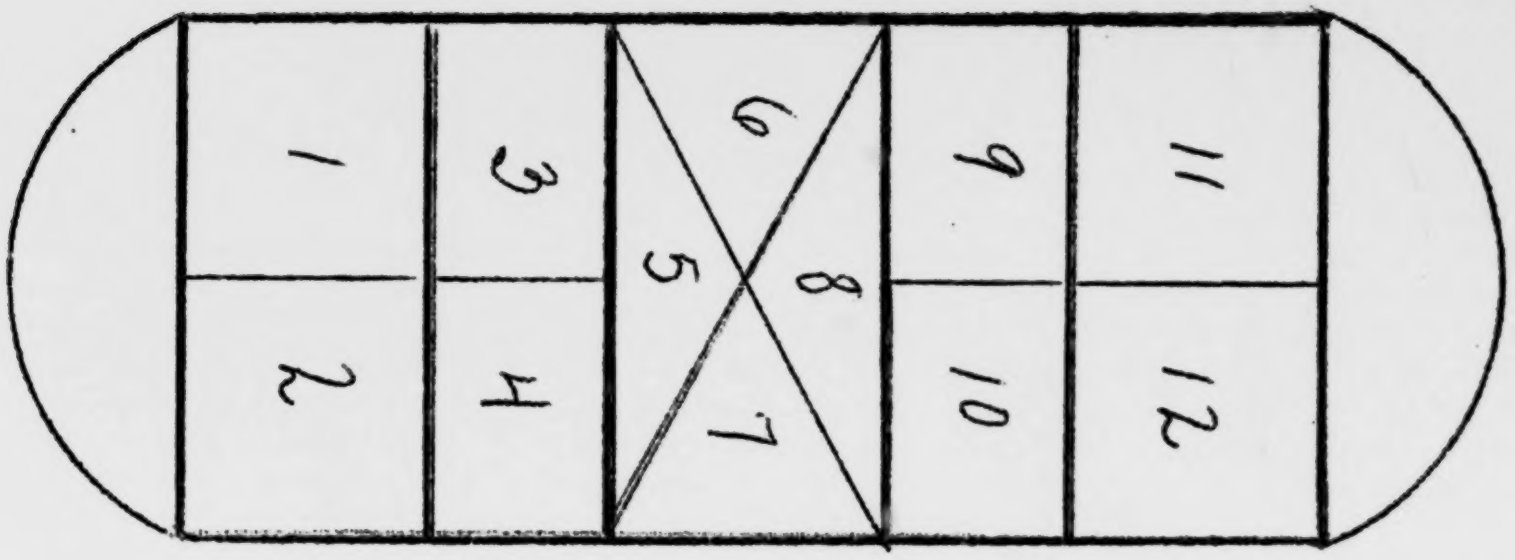


E.

Try these same multiples on every row and see what patterns develop.

HOPSCOTCH

MATHEMATICS



Like the number line direction-finder, a Hop Scotch painted on the playground will provide many hours of counting fun and number challenge.

Stations should be jumping or hopping size (11 and 12 are optional and may be added later, if desirable). Besides the standard Hop Scotch procedures, try getting more mileage from Hop Scotch with problems like these:

Beth and Martha were playing Hop Scotch. Beth had the first turn. One of them is in section 4. They each missed once on the same section. Who is in section 4?

What are the most turns each could have had?

Who has the next turn?

If Beth misses on one more section and Martha on two more sections, who will win?

-- Helen Strawn
Deep Run Valley

BULLETIN BOARDS that WORK!!! (and teach, too)

Your bulletin boards can be functional as well as decorative. A bulletin board that really works can be made with felt backing and rough-surfaced materials, as described previously. Boxes tacked in the bottom corners will serve to hold objects to be used.

Others can be made with construction paper background and objects, forms, ladders, etc., which can be attached with yarn, string, etc., and "operated" in various ways. Examples: adding machine, multiplication machine, special number lines, big clocks, etc.

A.

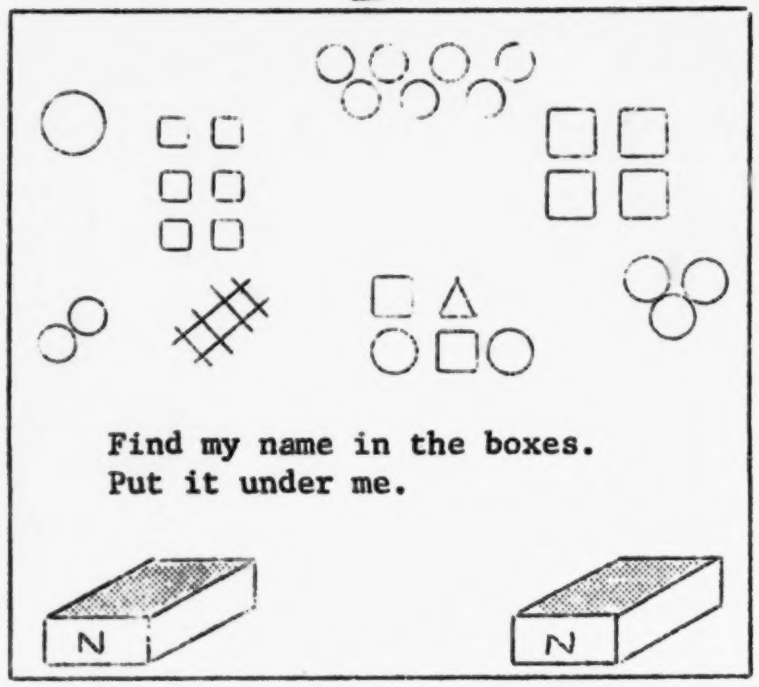
I am an
ADDING MACHINE
Stretch my lines across and up and down. See what happens!
6 7 8 9 10 11
5 6 7 8 9 10
4 5 6 7 8 9
3 4 5 6 7 8
2 3 4 5 6 7
0 1 2 3 4 5 6

Yarn or braided raffia can be attached with thumb tacks to make a movable number line as in A., above.

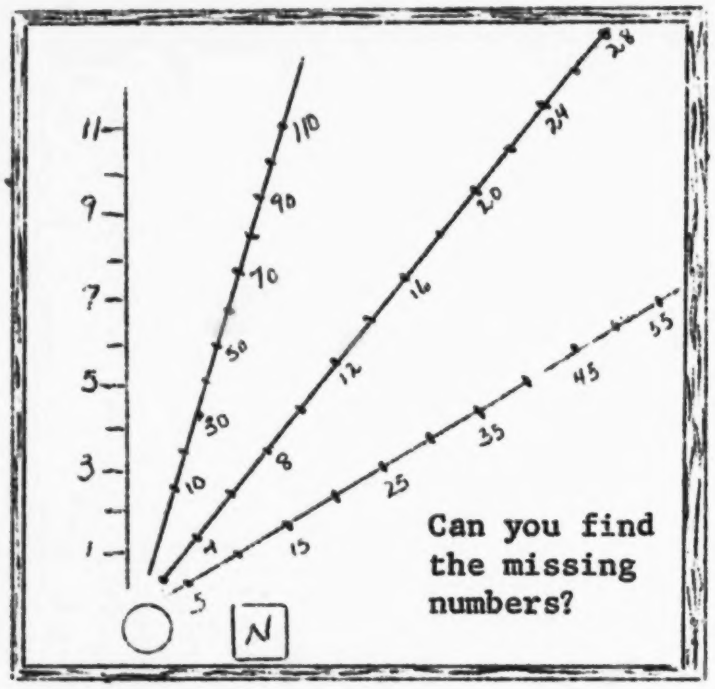
Boxes may be stapled or tacked on board to hold objects, shapes, or numerals. (B and C.)

Children enjoy planning a bulletin board that "works." See what's missing in C.

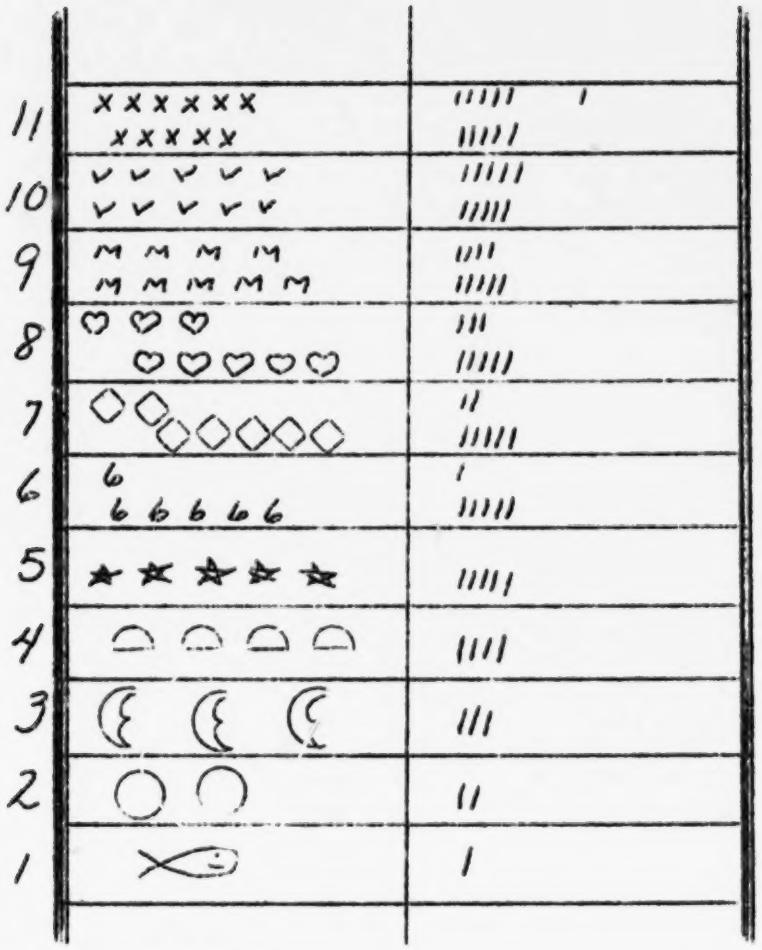
B.



C.



ANOTHER BULLETIN BOARD THAT "TEACHES" .....



Use number line going up a ladder and down a ladder to teach beginning numbers.

Materials - Bulletin Board or
Flannel Board
Construction paper or felt for ladder
Objects, tallies, and numbers

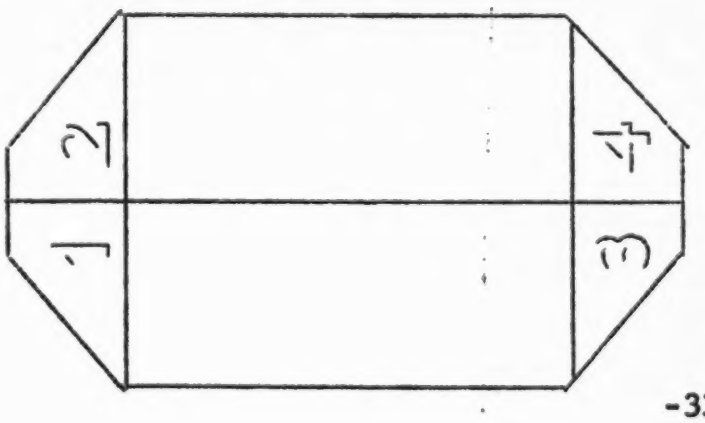
This same form could be made into an addition or subtraction ladder.

-- Minnie Peterson
Makefield, Pennsbury

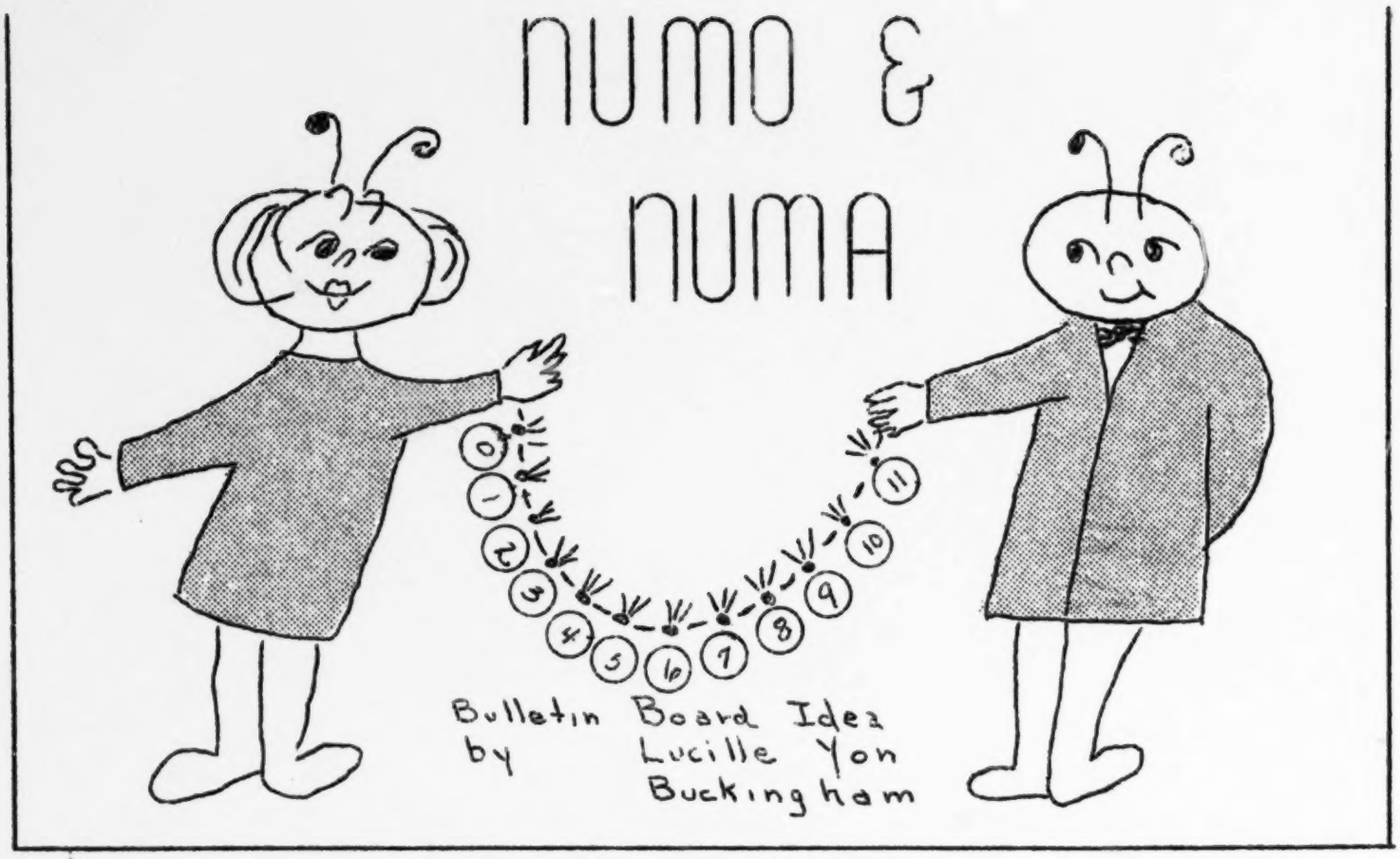
Use movable objects and tallies for children not ready for numerals. Children who are ready can use the objects for counting, regrouping (how many names for a number?), and for problem solving.

DESK SHUFFLEBOARD

Tape or paint a shuffleboard court on an extra desk. Use bottle caps or discs and "pushers." Four children may play at a time, two at each end of the miniature court. A fifth may keep score. Children take turns, alternating from each end. Numerals may be changed to vary the game and gradually make it a more challenging combination practice.



-- Lorrayne Brindley
Pennsbury



This manual would not be complete unless we mention Numo and Numa. They are the delightful, fantasyland characters who skip in and out of the Wirtz, Botel MATH WORKSHOP FOR CHILDREN as "Number Explainers" extraordinary.

These cute creatures captured the fancy of everyone who became acquainted with them at the August Elementary Mathematics Workshop.

Before the two weeks were over, they began appearing in the afternoon first- and second-grade sessions as hand puppets, jump rope holders, bus number tags, and RESEARCH CENTER supervisors. They were song and game instigators. Some of these ideas are presented on the following pages.

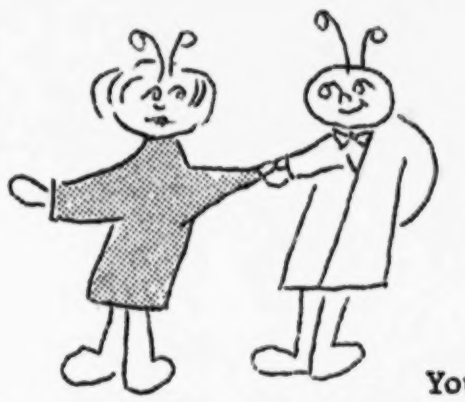
Sometimes they were made of felt; other times they appeared on cardboard or plywood. They would just stand around for awhile and all of a sudden, they'd appear on a bulletin board explaining some new mathematical idea.

We even heard a rumor that they may indicate the way to the areas usually referred to as "His" and "Hers" in some Bucks County classrooms this fall.

Two things are certain: they're going to be hard at work explaining arithmetic this year, and they "sparked" our workshop in a number of ways.

INTRODUCING NUMO & NUMA

I.



You will meet Numo and Numa in our first grade room on Sept. 7. They will help us have fun at work and play.

Mrs. Fox

Sara Fox, of Doylestown Township, sends a card to each of her prospective first graders a few days before school starts.

Here is this year's message.

II. Then she has a sign on her classroom door like this one.

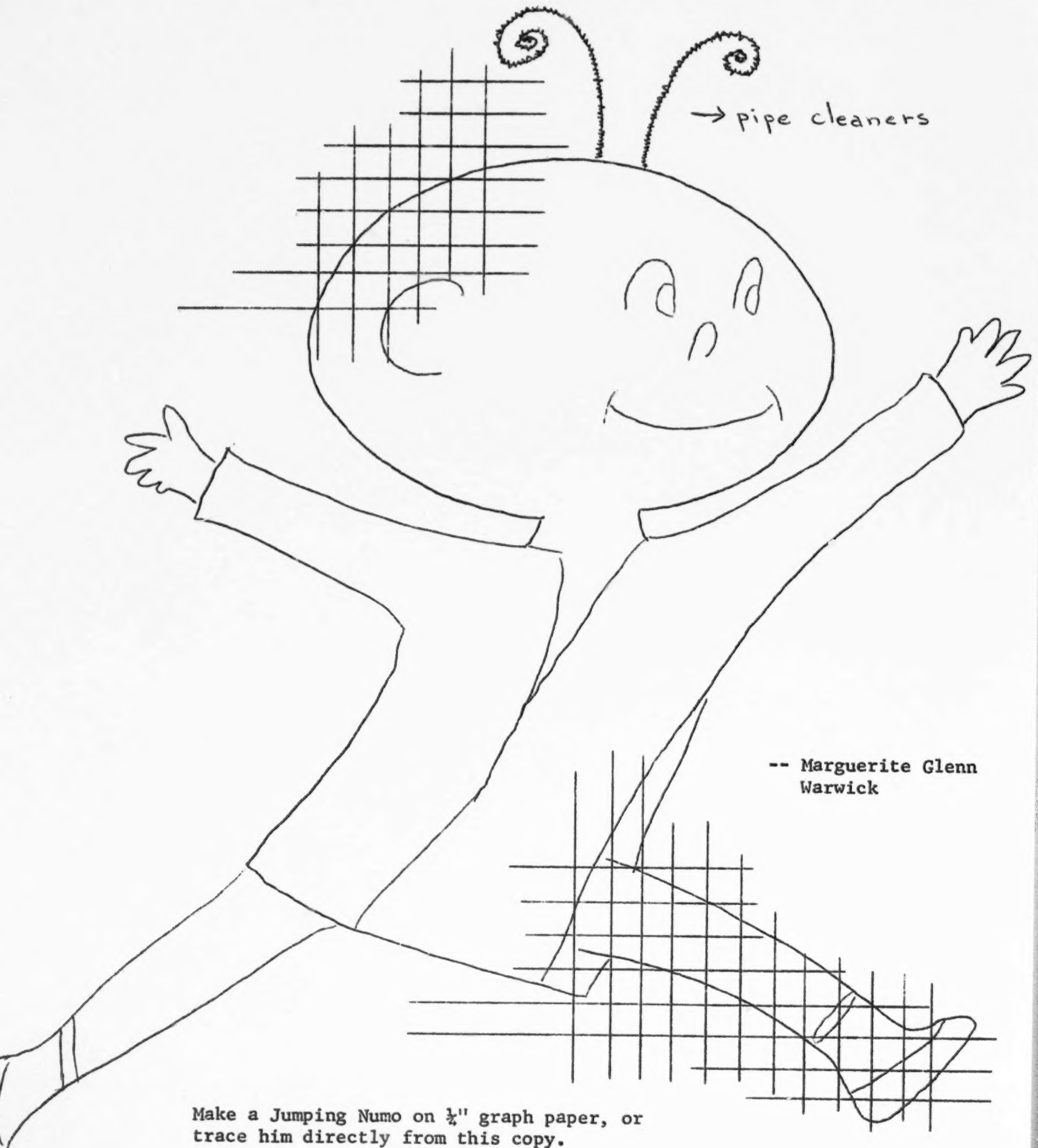
Meet me in First Grade.

Mrs. Fox

III.

On the next page you will find a pattern for a Jumping Numo, devised by Marguerite Glenn of Warwick. Note that Numo and Numa are ageless, timeless characters like Jimminy Cricket and Peanuts. The antennae have been invented by Wirtz, Botel to give them a "space age" look and to endow them with their extraordinary "number sense."

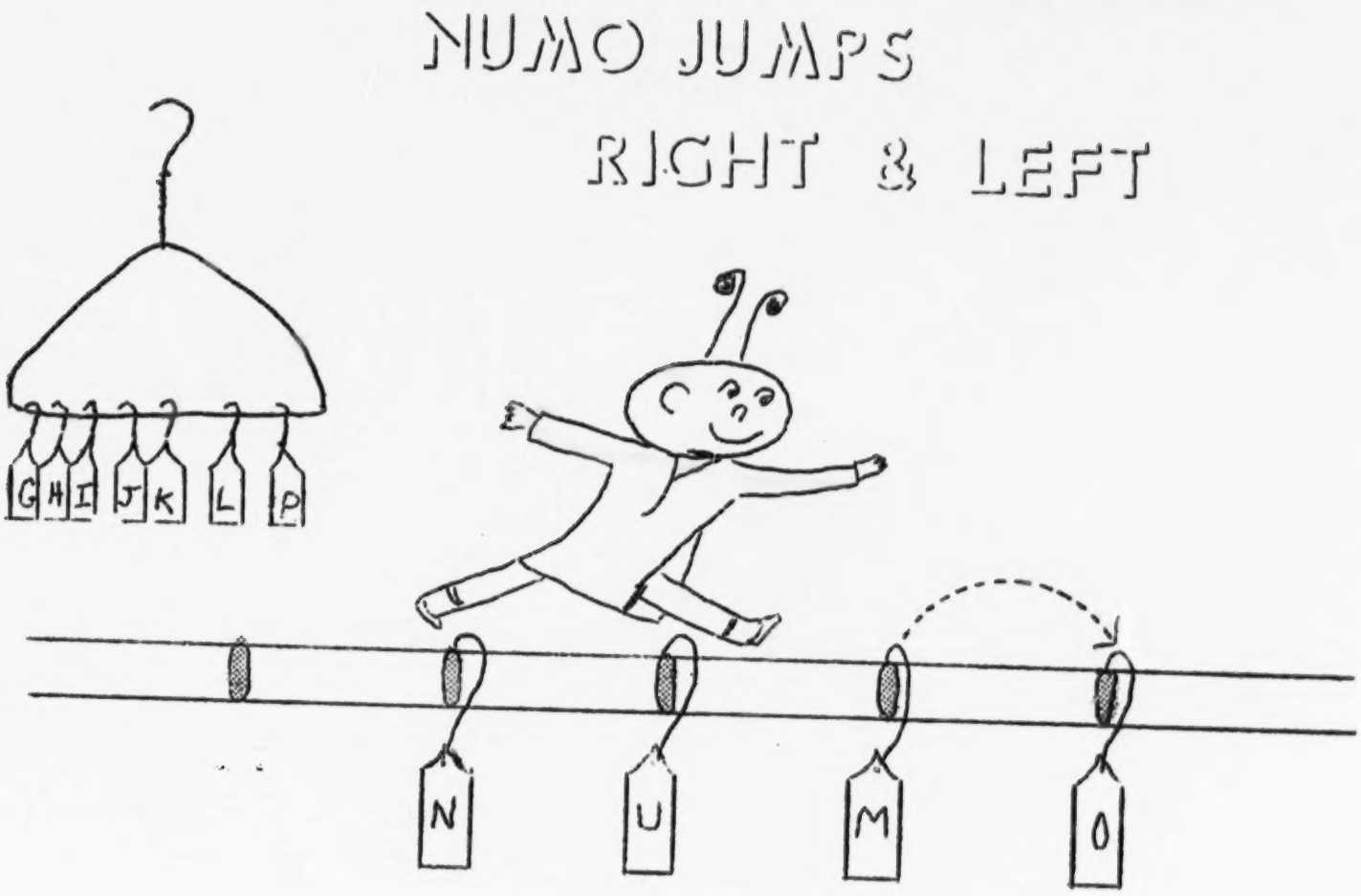
A PATTERN FOR JUMPING NUMO_____



-- Marguerite Glenn Warwick

Make a Jumping Numo on ¼" graph paper, or trace him directly from this copy.

See suggestions for use on next page.



The folding number line described previously can be tacked on a bulletin board. The Jumping Numo just illustrated becomes the jumping "explainer" when drawn and cut from cardboard. He can be colored and covered with clear plastic.

By hanging an N on a station, he can spell his name by "jumping one to the right, and hanging the second letter, U; one more jump and hanging M; and, finally, O."

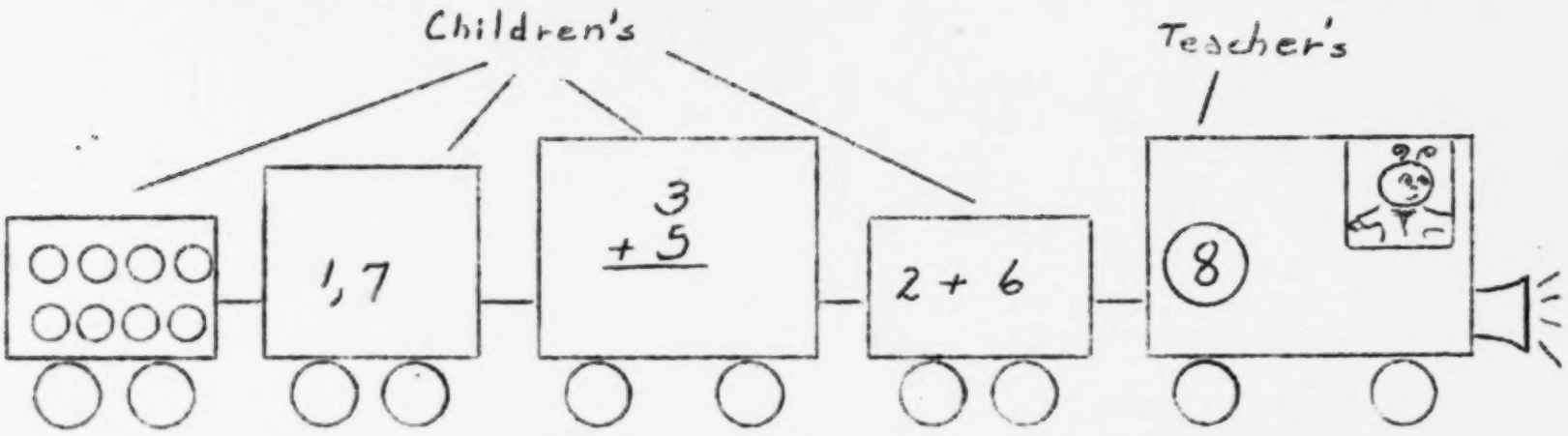
Numbers can be used to form a series, and spaces between stations can be larger jumps; for instance - 2, 4, 6, 8, 10; 3, 6, 9, 12, 15; 4, 8, 12, 16, 20; 5, 10, 15, 20, 25, etc.

Pupils can "jump" Numo - 2 ↷ or ↶ 2 . Others can follow his directions on the classroom floor number line, or with pencil and paper at desks. Use terms like right, left, forward, back, skip, and over.

-- Hannah Hampton
Wrightstown

NUMO'S HOUSE AND TRAIN

NUMBER NAMES



Ask children to make a car for Numo's train. Put up any example that's different. If the number is taped to the head car, it can be changed often and new cars will make the train.

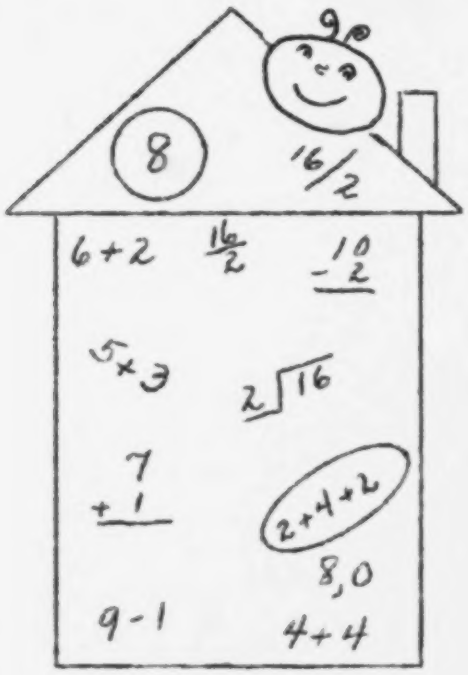
To vary the above activity for smaller groups:

Have children think of number names or different ways of saying 8, and have them "live" in number house, or Numo's house.

"How many names for the number ... have you found?"

"Who can think of another one?"

"Do you think there are more?"



Made from large colored "background paper." (24"x36")

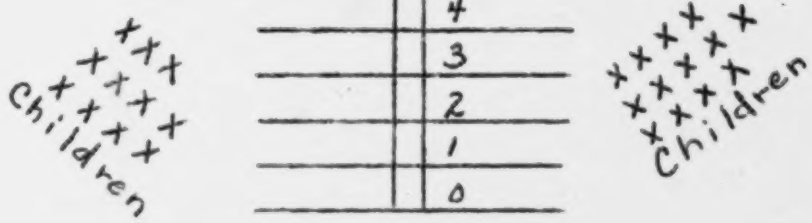
-- Jeanne Kryzwicki
Pennsbury

NUMO HAND PUPPETS

Children from 3 to 103 have a special fondness for puppets. Constructing a Numo Puppet to explain and develop the use of the number line will be most helpful in teaching, reviewing, and exploring new ideas and concepts in addition, subtraction, multiplication, and division. Comparisons can be made and understanding extended.

Materials Needed:

One-fourth yard white flannel, felt, or corduroy
Two 5" or 6" ovals stuffed and sewn together
One linen marker for facial features
Two pipe cleaners for antennae
Puppet stage - an overturned table will do
Number line,) to ...? (with pictures, dots, tallies, or numerals)



Suggested Uses:
Numo may call a child to "jump" on the number line. For instance, he might say: 'Sally, jump 2 spaces. Paul, jump 4 spaces. How many spaces did Sally jump? How many spaces did Paul jump? Did Sally jump as far as Paul did? Did Paul jump as far as Sally did? How much farther did Paul jump than Sally jumped?"

Encourage children to discover the answers. Let children whisper their answers to Numo; a nod will let them know whether he has given the correct response. If not, he returns to his place and reviews his thinking.

Variation:
The game of "Giant Steps" or "Mother, May I?" may become "Numo, may I?". Numo tells a child to take rabbit hops, giant steps, umbrella steps, etc., on the number line. Child must ask: "Numo, may I?". Then children count steps taken, how many spaces covered, etc. If they forget to ask, "Numo, may I?", they must return that number of steps, and thus "subtract" from their position on the number line.

-- Clara Stacer
Pennsbury

NUMO & NUMA SONGS & GAMES

Tune: "Galloping H[illegible]se," by Verna Meade Surer,
Ginn & Co., First Grade Music Book,
page 34

Oh, Mr. Numo can jump on the floor,
All day, all day, all day.
Oh, Mr. Numo can jump on the door,
Away, away, away.

This song might introduce Mr. Numo who could be represented by a puppet or a cut-out. He could jump, as the song is sung, on a number line on the floor, chalkboard, or up the wall or door frame.

Tune & Words: "Ten Little Indians"

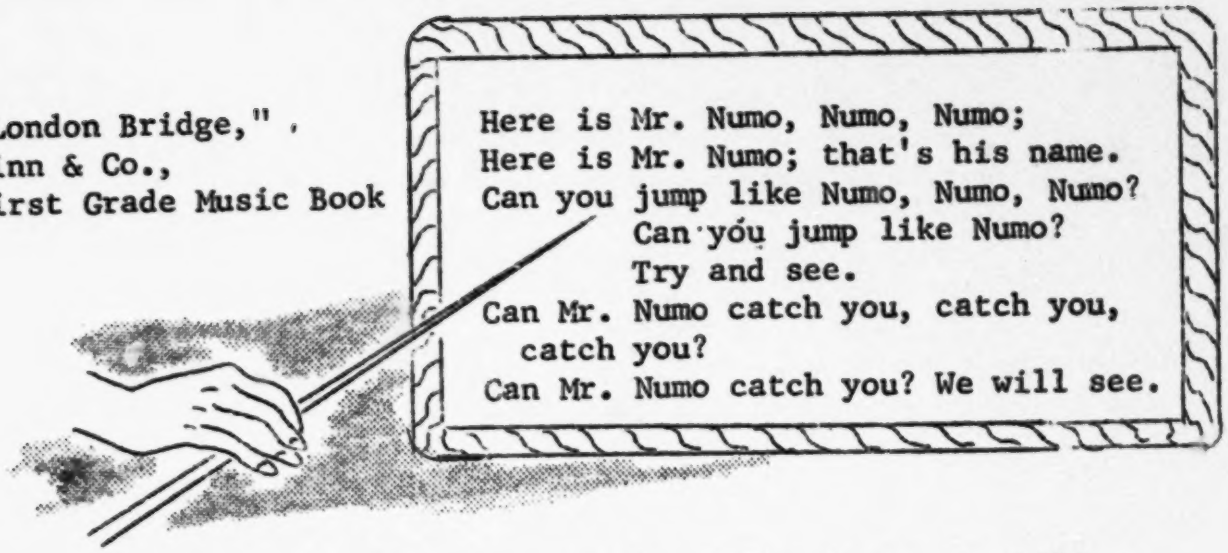
Distribute labels to the children. Make them in the shape of Numos, or have Numo painted on them and number from one to ten. In the song, substitute Numo for Indian. As the song is sung, the child with that number comes and stands in place on the number line. Sung in reverse, the children could leave the number line, one by one. Discuss: What happened when we came up and went away? How many went up in all? How many were left there as each one returned?

Tune: "This Old Man," English Singing Game
Ginn & Co., First Grade Music Book

Mr. Numo jumped one, he jumped one upon my thumb. With a nick-nack, paddy whack, give the dog a bone, Mr. Numo jumped back home. (Two-shoe; three-knee; four-floor; five-a hive; six-stick; seven-up in heaven; eight-gate; nine-spine; ten-again)

Mr. Numo can jump from number to number on a number line. Children can "be" Numa and Numo and jump, too. Students can tally jumps with fingers or tally marks.

Tune: "London Bridge," Ginn & Co., First Grade Music Book


Here is Mr. Numo, Numo, Numo;
Here is Mr. Numo; that's his name.
Can you jump like Numo, Numo, Numo?
Can you jump like Numo?
Try and see.
Can Mr. Numo catch you, catch you, catch you?
Can Mr. Numo catch you? We will see.

Make circle formation of small groups or the whole class. A child is chosen in each circle to be Numa or Numo. The children sing, and Numa or Numo stands in the center of the circle with his hands on his head for antennae. Children sing and jump.

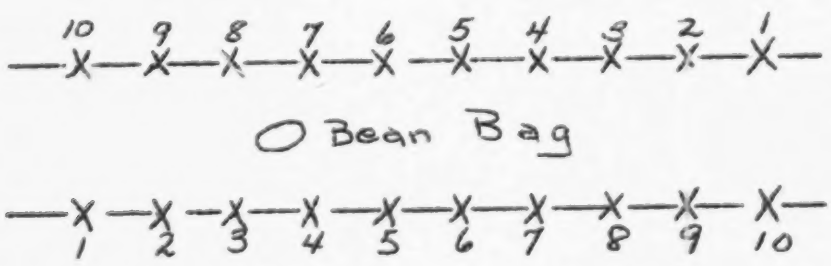
In the third verse, Numo or Numa choose someone to chase by standing in front of the child they have chosen. After singing, they proceed to race around the outside of the circle until "caught" or "home." If caught, they become Numa or Numo.

A Game

Children stand on two number lines facing each other. One line is numbered from 0 to 12, the other from 12 to 0, as shown below.

There are two teams: Team 1 - Numa's, and Team 2 - Numo's. Each child stands on a number and represents that number. First the teacher, and later another child, calls a combination fact. The two children who have this "sum" as their number run to the center to get Mr. Numo (a bean bag version). The child who gets him first and returns to the line, gets a point for his side. If children standing on the correct solution to the combination "fact team" make the score, their team gets one point. If they ran to the center incorrectly, their team looses a point. A scorekeeper keeps a record.

This could be varied in a number of ways and could employ addition, multiplication and division facts, as well as letters which combined make two, three, or four letter words.



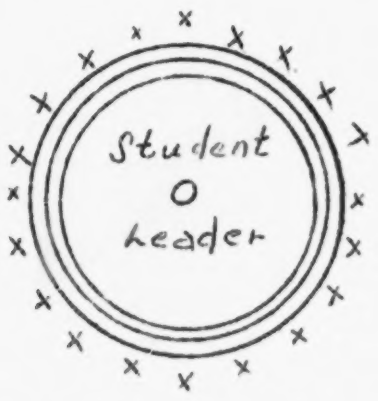
-- Alverta McGarity
Pennsbury

Do They REALLY Understand What You Say?

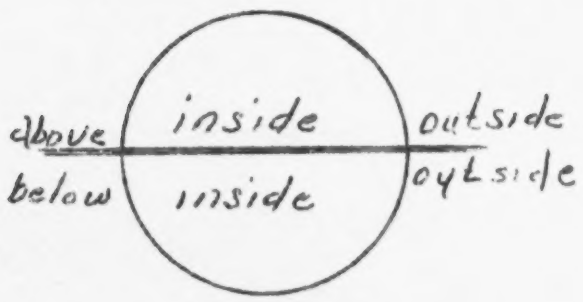
All primary school children need continued help with <u>really listening</u> and understanding directions and instructions. It is easy to assume that they know terms and associate specific action with these terms. There are some selected words they should know and understand well; to comprehend the confusion which exists, just ask any group of first or second graders to locate the <u>top left hand corner</u> of his paper - or the <u>bottom right hand corner</u> of a page. You will find that there is much confusion about such terms as: <u>left</u>, <u>right</u>, <u>top</u>, <u>bottom</u>, <u>opposite</u>, <u>more than</u>, <u>less than</u>, <u>above</u>, <u>below</u>, <u>inside</u>, <u>outside</u>, <u>large</u>, <u>small</u>, etc.

Use these words often in giving instructions to your students so that they become everyday familiar words which are interpreted correctly.

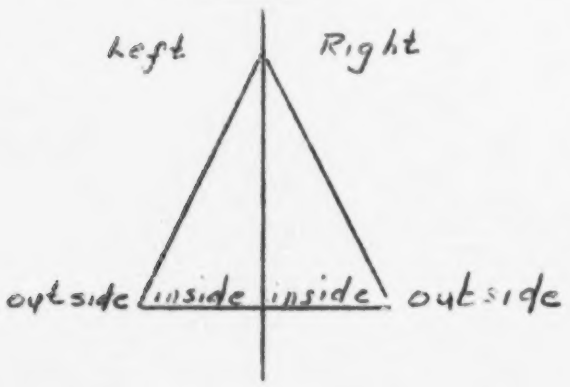
The action song - "I put my left foot in,
I put my left foot out,
I give myself a shake, shake, shake,
And turn myself about" -
can be used advantageously, especially if one has the children stand on a circle taped on the floor or painted on the playground. Add <u>left hand</u>, <u>right foot</u>, <u>right hand</u>, <u>head</u>, <u>whole self</u> to continue the lyrics and reenforce understanding of "what's happening."



This same circle could have a line drawn horizontally to establish the following ideas:

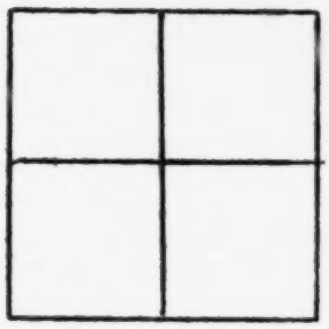


How many objects (or children) <u>above the line inside</u> of the circle? <u>above the line outside</u> of the circle? <u>below the line inside</u> of the circle? <u>below the line outside</u> of the circle? (or triangle, or square, or rectangle?) In all?



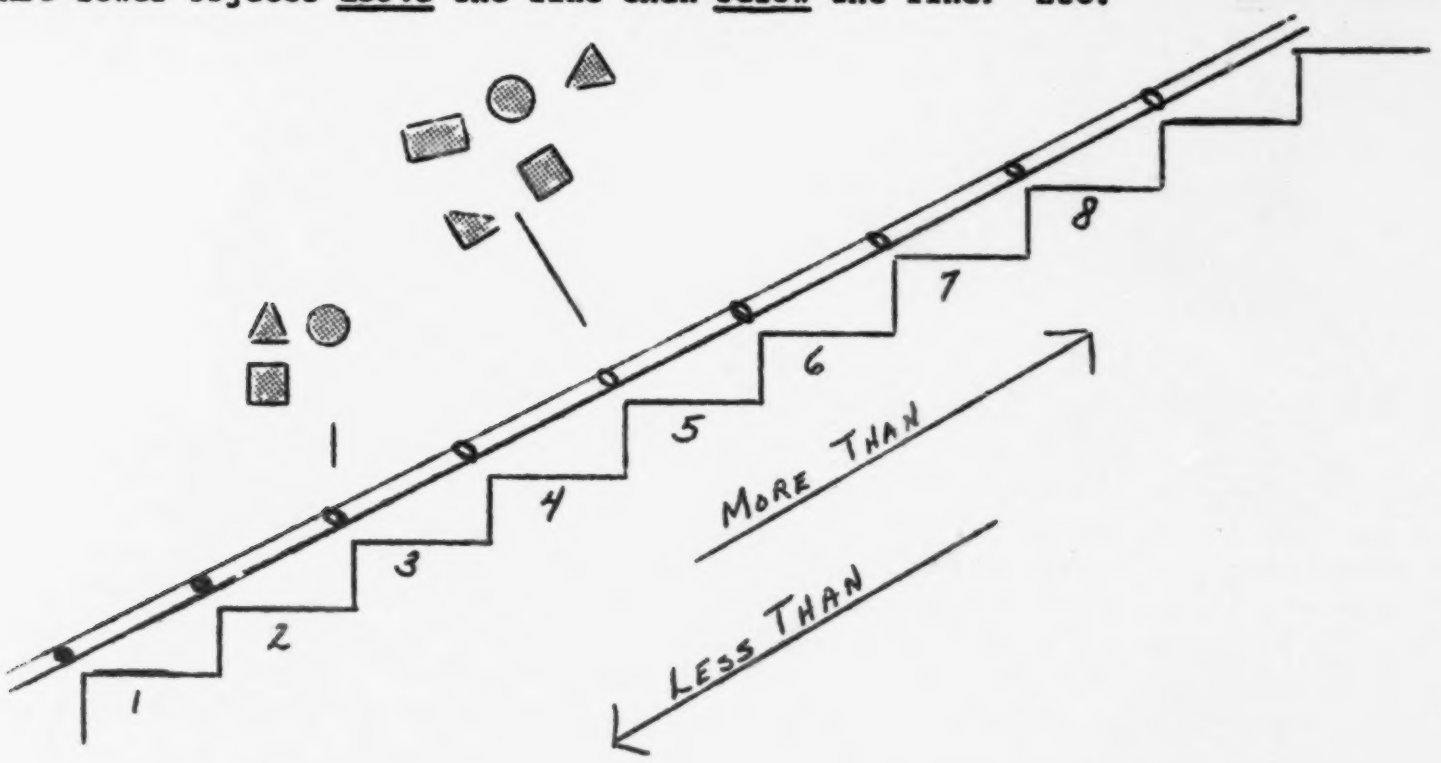
How many "things" on the <u>left side</u>? <u>right side</u>? <u>outside</u> of the triangle? <u>inside</u> of the triangle? <u>inside</u> of the triangle on the <u>left</u>? on the <u>right inside</u> of the triangle? <u>outside</u> of the triangle on the <u>right</u>? on the <u>left</u>? in all <u>inside</u>? in all <u>outside</u>? All together?

Do They REALLY Understand ...



Place 3 red objects (or write number 3) in the top left square ... 2 in the top right square ... 1 in the bottom left square and 4 in the bottom right square. How many in the top half? bottom half? All together?
Give similar instructions using a square of paper folded once sidewise and once lengthwise.

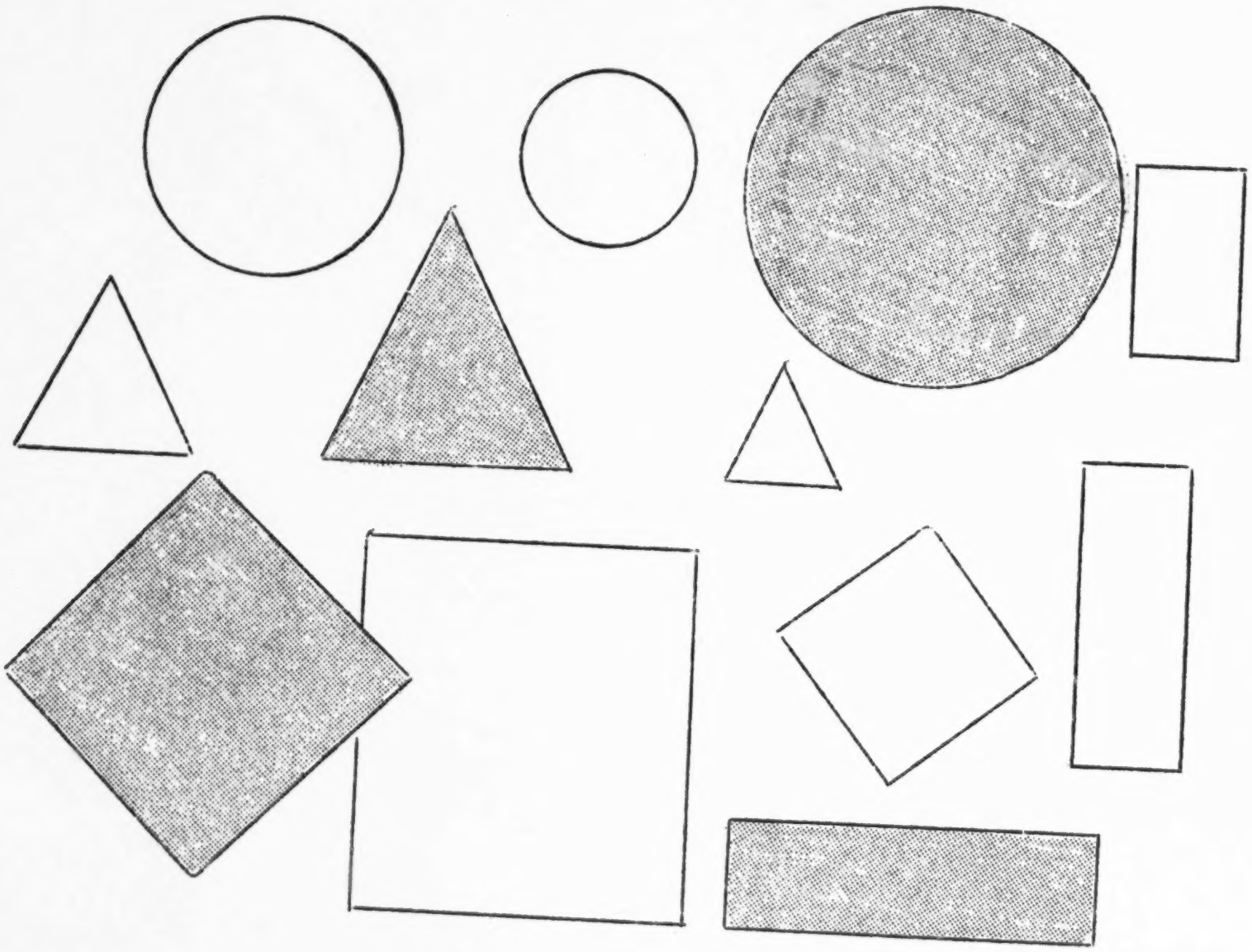
Use the same forms to help develop the idea of more than and less than. Are there more objects in the top half than there are in the bottom half? above than below? on the left than on the right? Which part has the least objects? Are fewer objects above the line than below the line? Etc.



How many more than 3 is 5? How many less than 5 is 3? Who can show us a number more than 6? less than 2? Etc.

Use these kinds of experiences often at work and at play - check to see that your students really hear, understand, and act accordingly. We can scarcely overdo this particular kind of "study skill" at the primary level.

LARGE, SMALL, & IN-BETWEEN



Right from the very beginning, include different shapes, sizes, and colors in your sorting boxes, flannelboard, and magnetic board materials. They develop rich creative and critical thinking and children are fascinated with them. See the next page for several possible uses.

Encourage children to match by color, shape, and size. Let them arrange squares, circles, triangles and, later, numbers by size. Develop vocabulary with proper names for the shapes. (Next page.)

Cut some shapes into halves, thirds, quarters, etc. Can they find the matching half or piece? Would they balance on a mobile? Let them try similar and different shapes.

Let each child keep a scrapbook in which to paste construction paper materials according to an agreed-upon rule. He may have a place for "mirror images," discoveries about specific shapes and numbers, drawings of mobiles that "work" and those which do not work. Some children enjoy keeping this kind of a record.

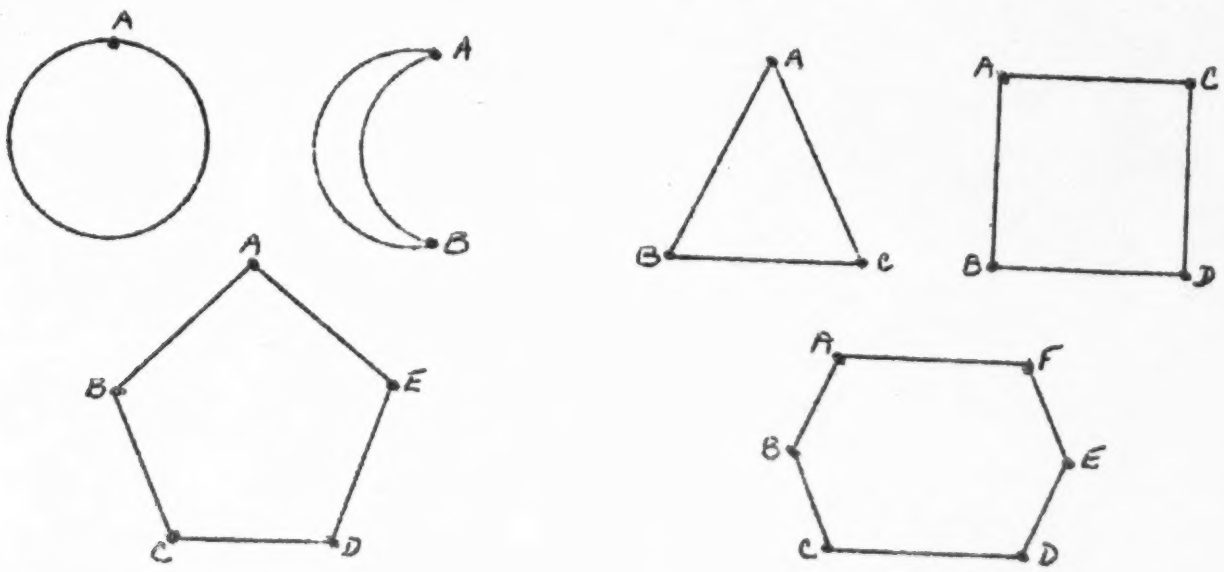
MORE ABOUT SHAPES

Geometry is one of the unifying principles of mathematics. Through using and observing various shapes, sizes, and lines right from the beginning, young children are using big mathematical ideas long before they ever have a name or a rule.

Include shapes (like these pictures) in your Sorting Box collection. There are many ways to describe them, but children will like the "real words" for them: circle (a line that goes all the way around and has no corners); triangle (any 3 sides and 3 points); square (4 equal sides and 4 points); rectangle (4 sides and 4 corners with opposite sides equal and 2 sides longer or shorter); pentagon (any 5 sides and 5 points); hexagon (any 6 sides and 6 points).

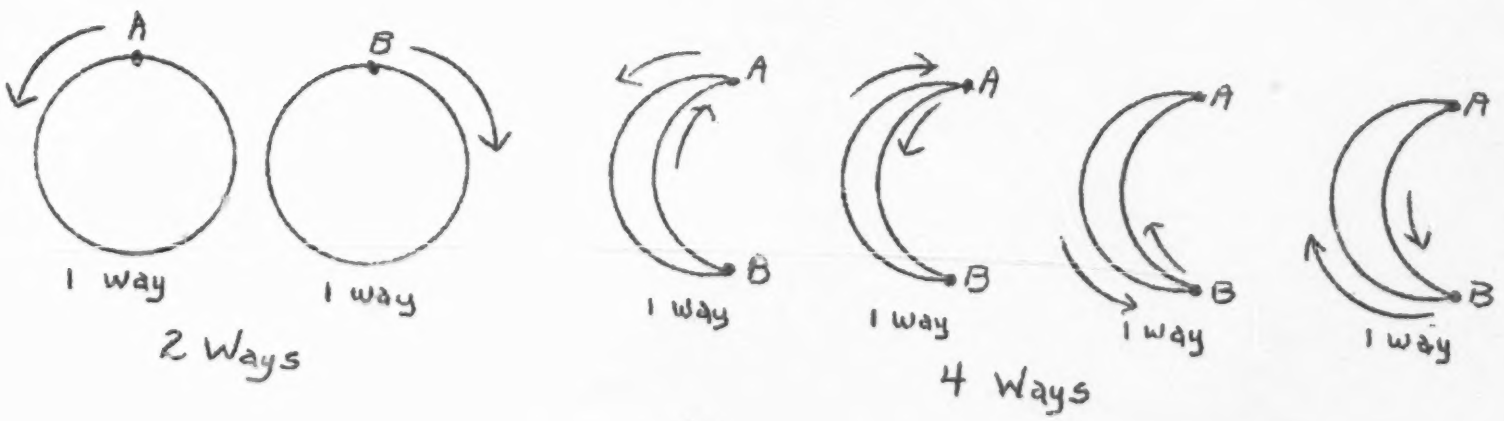
When children have had more experience with shapes, give groups of children a set like these. Let them use their fingers or a piece of paper to make some amazing discoveries and keep some interesting records of these discoveries.

Draw the following series of lines and shapes on the board:



Ask: <u>Do you recognize any of these shapes?</u>

Ask these kinds of questions: <u>How many ways could we draw the circle, starting at A, without taking our pencils off the paper?</u> (Two - left to right and right to left.) <u>The moon?</u> (Four - 2 clockwise and 2 counter-clockwise, starting at A the first time and B the second time.)



and MORE about shapes .....

Continue this same line of thought and questioning, letting your students try it at their desks or on the board. Do you think you have thought of all the ways? Let's see ... What have we discovered so far? Now we start a table, an important part of developing real mathematical thinking used extensively throughout the MATH WORKSHOP FOR CHILDREN. The question you will ask most often is: How can we keep a record? A record of this activity might look like this so far:

Lines	Ways
/	//
//	////
///	//////

Lines	Ways
1	2
2	4
3	? (Discover 6)

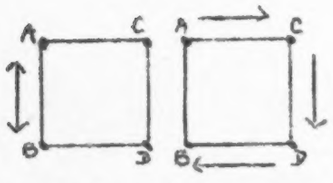
Now, what about the square? (Starting at each point and working clockwise and counter-clockwise, there are eight ways.) Do you think you could guess what would happen with the next picture which has five sides and five points? What about the one with six sides and six points? If the pattern, or "secret," has not been discovered yet, let children try working it out. Your completed record of this activity will look like this, and children have delved right into the heart of the magic of mathematics!

/	//
//	////
///	//////
////	///////
/////	//////////
//////	//////////
/////	?
//////	?

Lines	Ways
1	2
2	4
3	6
4	8
5	10
6	12
7	?
8	?

What's happening here? Can you say for sure how many ways you could draw a 7-sided figure? 8-sided?

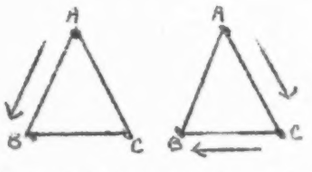
Besides finding the number of possibilities for making figures, you can ask your students to: Find the long way and short way to take trips from point to point on any figure. For instance:



A-B(short) A-C-D-B(long)

1 jump 3 jumps

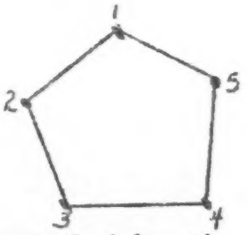
Total - 4 jumps



A-B(short) A-C-B(long)

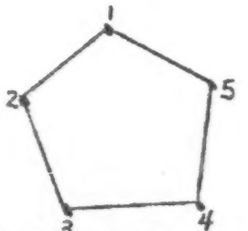
1 jump 2 jumps

Total - 3 jumps



1-2 (short)

1 jump



1-2-3-4-5(long)

4 jumps

Total - 5 jumps

MULTI-CHALLENGE PROBLEMS

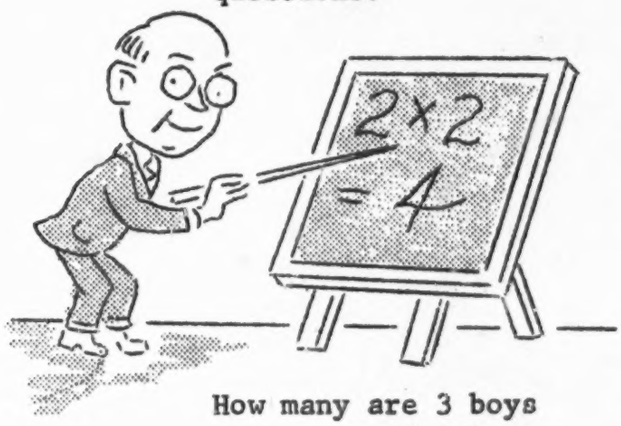
Drill does not have to be rote learning, nor does it have to be the end of an experience once the fact has been learned. This is one of the basic differences between traditional and contemporary approaches in all curriculum areas, and it is especially true in mathematics.

It is easy to ask questions which require a simple, "pat" answer. We can also ask impossibly difficult questions or "trick" questions which confuse thinking.

There is a happy medium; these questions require thinking, get the student actively involved, have answers which suggest other possibilities, and "take your somewhere beyond" an answer. The following questions illustrate this basic difference.

Questions which go somewhere:

Traditional questions:



1. Six children live within five blocks of the school. Do we know for sure that at least two of them live in the same block?

How many are 3 boys and 2 boys ? boys

2. Mother is putting John's shirts in his bureau. There are five drawers and six shirts. Do we know for sure that there will be at least two shirts in one drawer?

John has 6 shirts. Tom has 5 shirts. How many more shirts has John than Tom?

3. Jane and Bill went to the shoe store with Mother. They bought three pair of shoes. Do we know for sure that one child has more than one pair of shoes?

How many shoes are there in 3 pair?

4. There are five theaters in town. How many children must go to the theater in order to be sure that there are at least two children in each theater?

Add:

4	5	3	5
+5	+5	+5	+3

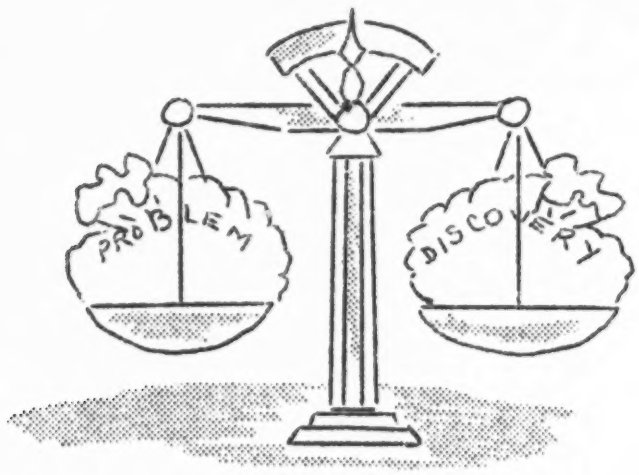
5. Four children went to the store to buy two pencils each. There was a sale: 10 pencils for 25¢. How many children got at least 3 pencils each?

Subtract:

8	9	10	9
-3	-4	- 5	-5

MULTI-CHALLENGE PROBLEMS (cont.)

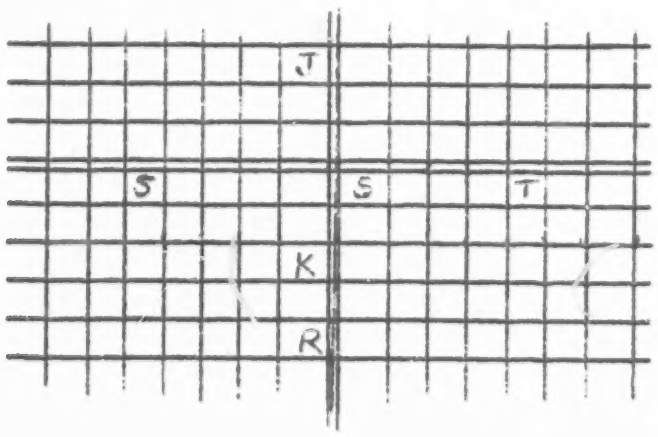
You can get a lot of mileage from a simple problem idea. In their simplest form, we could probably put a "grade level tag" on them. Just think how much more interesting they can become just by extending them to encourage real thinking and challenge. Try some of these and see what your students do with them. Use the story headline technique as you and your students create problems, and ask the kinds of questions just suggested.



Several children signed up for a new book. Kathy was 8th on the list. If four names are crossed out, how many people will get the book before Kathy does? If one of these keeps the book for 3 days and each of the others keeps it 2 days, how many days must Kathy wait for the book? If each keeps the book 1 day? 3 days? 2 and one-half days?

$3 + \square = 8$

Richard stops for Karen each morning and they walk to school together. Karen lives 3 blocks from school and Richard lives 5 blocks from school. How many blocks does each walk to get to school? Who walks farthest? How much farther? How many blocks does Karen walk to and from school? Richard? If Karen goes home for lunch, how far does she walk in a day? Richard? How many would they both walk if they both went home for lunch? Suppose Karen moves. Suppose Richard moves. Suppose they walk home with Johnny, Tommy, or Susan some night?



-- Helen Strawn
Deep Run Valley

MULTI-CHALLENGE PROBLEMS (cont..)

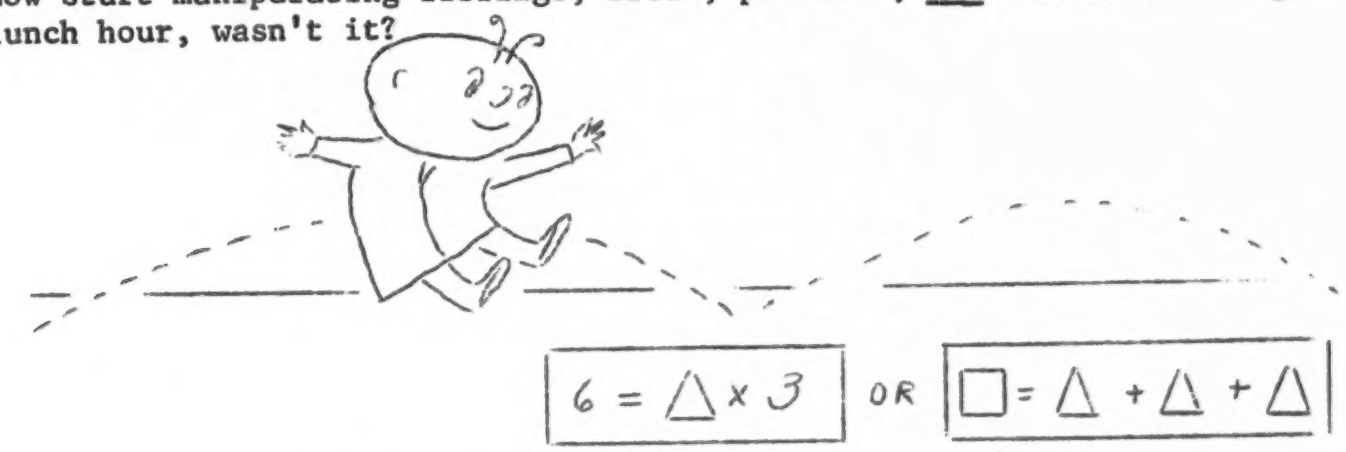
Mrs. Smith packs 3 lunch boxes every day. She puts two sandwiches in each lunch box. How many sandwiches must she make? She puts one white and one whole wheat sandwich in each lunch box. How many white and how many whole wheat must she make?

Monday, she made chicken sandwiches and tuna fish sandwiches. George doesn't like tuna fish, so she made his sandwiches of chicken. She made Jane and Mike one sandwich of each kind and George two of the same kind. How many chicken sandwiches did she make? How many tuna fish?

One day, George wanted to take enough lunch for a friend. If he took two extra sandwiches that day, how many sandwiches did Mrs. Smith make? Jane and Mike decided they wanted only white bread, but George wanted one white and one whole wheat sandwich each for himself and his friend. How many white and how many whole wheat sandwiches did Mrs. Smith make?

The Smith children decided their lunches were too large. George decided to have one and one-half sandwiches, Jane one sandwich, and Mike one and one-half sandwiches. How many did Mother make? Mike and George wanted half of a whole wheat sandwich each. How many whole wheat and how many white did Mother make? How many halves?

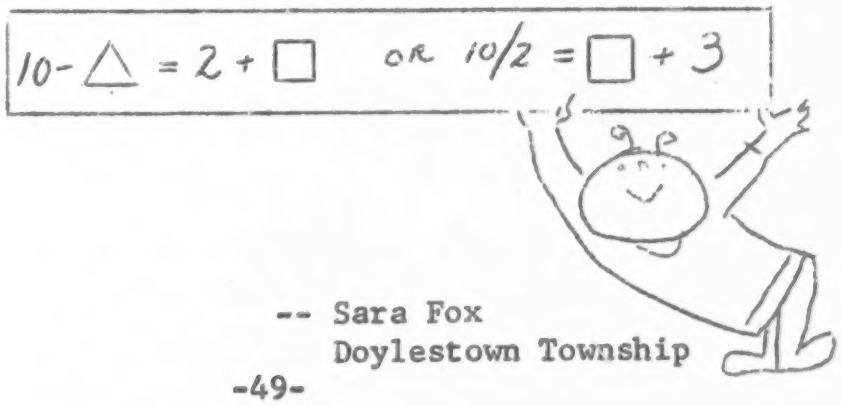
Now start manipulating fillings, bread, portions, and children. Long lunch hour, wasn't it?



The Jones family and the Smith family went to the shoe store. There were free balloons for each child. The Jones family got three balloons. The clerk gave the two families five balloons in all. How many children in the Smith family?

If each child got one pair of shoes, how many shoes did the two families buy?

If two children got two pair of shoes, how many pairs did they buy all together? How many shoes did they buy? If one pair were put in each box, how many boxes of shoes were there?



-- Sara Fox
Doylestown Township

based on <u>number of children</u> in classroom

1. How much money will we need to buy a popsicle for everyone in the class? How many different ways can we make payment using pennies, nickels, dimes, quarters, half dollars, and dollars? (Vary combinations which may be used in making payment.)

? ? ? ? ? ?

2. We buy rope by the hank. There are 100 feet in a hank. It is coiled so that it can be cut in half.

   It takes 8 feet to make a single jump rope, and 12 feet to make one for a group.

   How much rope do we need to make a jump rope for every girl in the class? For every girl and four long ones besides? For all boys? For all boys and girls? For all members of the class and five long ones?

   If we buy a hank and cut a single rope for every two girls, how many big ropes can we cut? How many can we make from a hank if we make all singles? All big ropes?

   If we make small ropes 9 feet long and big ropes 15 feet long, how many of each can we get from a hank?

   If rope is $1.50 a hank, how much will it cost to buy two hanks? one and one-half hanks? two hanks? Etc.

8 15 4 9

3. If all the boys play kickball, how many players will there be on each side? Will there be an even number or an odd number on each side? How many ways can you find to arrange the players? If two boys are absent, how many ways are there to arrange the players? If only ten boys play, how many will there be on each side? How many different ways can you find to form the team if there are 12 boys? 14 boys? 15 boys?

-- Helen Strawn
Deep Run Valley

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